MA0004 Mathematical Analysis 1

4th Seminar

Real function of a real variable and its derivative

A. Geometrical Interpretation of Derivative

Inquiry-based task

Input the function $f(x)=\frac{3x-1}{2x+3}$ in Geogebra Online. You can use your mobile phones or tablets.

1. Using Geogebra compute $S1=f(4), S2=f(-1)$.
2. Draw the line s intersecting the points S1, S2.
3. The slope of the line $y=x-3$ is equal to 1. Compute the value $k=\frac{f(4)-f(-1)}{4-(-1)}$ and compare it with the slope of the s.
4. Create a descending slider a starting at 4 and terminating at -1. Set up the point S1 so its coordinates correspond to the slider a. [S1 = (a, f(a))]
5. Modify the slope k definition and substitute the value 4 by the value a of the slider.
6. Start the animation. Follow the changes in the parameter k, the line s and its equation.

Final discussion on the definition of some function's derivative and how it is connected with the animation and parts of the figure.

B. Differentiation using the elementary formulas

Find a derivative of the following functions:

1. $f(x)=\frac{x^2\cdot\sqrt[3]{x}}{\sqrt{x}}$

2. $f(x)=\frac{x+\sqrt{x}+1}{\sqrt{x}}$

3. $f(x)=x^2\cdot\ln x$

4. $f(x)=\frac{x^2+1}{x^2-1}$

5. $f(x)=\frac{1+\sin x}{\cos x}$

C. Differentiation of composite functions

Find a derivative of the following functions:

1. $f(x)=\sin^4 x$

2. $f(x)=e^{x^2-2x+1}$

3. $f(x)=\ln^3\left(x^2-1\right)$

4. $f(x)=\mathrm{tg}^3 2x$

5. $f(x)=5^{x^2-1}+3$

6. $f(x)=x^2\cdot\sqrt{1+x^2}$

7. $f(x)=\dfrac{1}{(5-2x)^2}$

8. $f(x)=\mathrm{arctg}\dfrac{1+x}{1-x}$

D. Special modification before differentiation

Find a derivative of the following functions:

1. $f(x)=x^x$

2. $f(x)=x^{\ln x}$

3. $f(x)=x^{\sin x}$

E. Tangent line and Normal line

1. Write the equation of the tangent line and normal line for the function $f(x)$ instersecting the point $T=[x_0,y_0]$.

a) $f(x)=\dfrac{3x-1}{2x+3}, T=[2,?]$

b) $f(x)=\dfrac{2x^2-1}{x+1}, T=\left[-\dfrac{1}{2},?\right]$

c) $f(x)=\dfrac{8}{x^2+4}, T=[2,?]$

d) $f(x)=x\cdot\ln x, T=[e,?]$

2. Write the equation of the tangent line and normal line

a) for the circle $x^2+y^2=2$ in its point $[1,-1]$

b) for the parabola $y^2 = x$ in its point $[4,-2]$