MA0004 Mathematical Analysis 1

3rd Seminar

Real Function of a real variable and its limit

Inquiry-based task

1. Make groups of 2-4 people. One of the group specifies limit conditions or requirements on continuity of an unknown function $f(x)$. The others try to find an example of the function which meets the requirements. You can change the roles then.

Examples of requirements:

a) Find the function $f(x)$ such that $\lim_{x\to 3} f(x) = 5.$

b) Find the function $f(x)$ such that $\lim_{x\to 3} f(x) = 5$, but $f(x)$ is not continuous for $x = 3.$

c) Find the function $f(x)$ such that $\lim_{x\to 0} f(x) = -\infty.$

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2. Try to use elementary modifications to solve the following limits:

a) $\lim_{x\to -1} \frac{x^2+4x+3}{x^3+1}$

b) $\lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49}$

c) $\lim_{x\to 0} \frac{\sin 2x}{3x}$ [we know that $\lim_{x\to 0} \frac{\sin x}{x} = 1$]

d) $\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$

e) $\lim_{x\to -\infty} \left(4x^3 - x^2 + x + 2\right)$

f) $\lim_{x\to \infty} \frac{2x^3 - x^2 + 5}{x^2 + x - 2}$

g) $\lim_{x\to \infty} \frac{\sqrt{x} - 6x}{3x+1}$

h) $\lim_{x\to\infty}(\sqrt{x-2}-\sqrt{x})$

i) $\lim_{x\to 1}\dfrac{x+1}{x^2-3x+2}$

j) $\lim_{x\to 0}\dfrac{1}{x^3-x^2}$