## **MA0004 Mathematical Analysis 1**

#### 4th Seminar

Real function of a real variable and its derivative

### A. Geometrical Interpretation of Derivative

### **Inquiry-based task**

Input the function  $f(x) = \frac{3x-1}{2x+3}$  in Geogebra Online. You can use your mobile phones or tablets.

- 1. Using Geogebra compute S1 = f(4), S2 = f(-1).
- 2. Draw the line s intersecting the points S1, S2.
- 3. The slope of the line y = x 3 is equal to 1. Compute the value  $k = \frac{f(4) f(-1)}{4 (-1)}$  and compare it with the slope of the s.
- 4. Create a descending slider a starting at 4 and terminating at -1. Set up the point S1 so its coordinates correspond to the slider a. [S1 = (a, f(a))]
- 5. Modify the slope k definition and substitute the value 4 by the value a of the slider.
- 6. Start the animation. Follow the changes in the parameter k, the line s and its equation.

Final discussion on the definition of some function's derivative and how it is connected with the animation and parts of the figure.

# B. Differentiation using the elementary formulas

Find a derivative of the following functions:

$$1. f(x) = \frac{x^2 \cdot \sqrt[3]{x}}{\sqrt{x}}$$

$$2. f(x) = \frac{x + \sqrt{x} + 1}{\sqrt{x}}$$

$$3. f(x) = x^2 \cdot \ln x$$

$$4. f(x) = \frac{x^2 + 1}{x^2 - 1}$$

$$5. f(x) = \frac{1 + \sin x}{\cos x}$$

### C. Differentiation of composite functions

Find a derivative of the following functions:

$$1. f(x) = \sin^4 x$$

$$2. f(x) = e^{x^2 - 2x + 1}$$

$$3. f(x) = \ln^3(x^2 - 1)$$

$$4. f(x) = tg^3 2x$$

$$5. f(x) = 5^{x^2 - 1} + 3$$

6. 
$$f(x) = x^2 \cdot \sqrt{1 + x^2}$$

$$7. f(x) = \frac{1}{(5-2x)^2}$$

$$8. f(x) = \arctan \frac{1+x}{1-x}$$

### D. Special modification before differentiation

Find a derivative of the following functions:

$$1. f(x) = x^x$$

$$2. f(x) = x^{\ln x}$$

$$3. f(x) = x^{\sin x}$$

### E. Tangent line and Normal line

1. Write the equation of the tangent line and normal line for the function f(x) instersecting the point  $T = [x_0, y_0]$ .

a) 
$$f(x) = \frac{3x-1}{2x+3}$$
,  $T = [2, ?]$ 

$$(b) f(x) = \frac{2x^2 - 1}{x + 1}, T = \left[ -\frac{1}{2}, ? \right]$$

$$c) f(x) = \frac{8}{x^2 + 4}, T = [2, ?]$$

$$d) f(x) = x \cdot \ln x, T = [e, ?]$$

2. Write the equation of the tangent line and normal line

a) for the circle 
$$x^2 + y^2 = 2$$
 in its point  $\begin{bmatrix} 1,-1 \end{bmatrix}$ 

b) for the parabola  $y^2 = x$  in its point [4,-2]