PLATINUM

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PLATINUM PROJECT

FINAL REPORT INTELLECTUAL OUTPUT (IO6)

GUIDELINES AND RECOMMENDATIONS FOR QUALITY ASSESSMENT IN INQUIRY BASED LEARNING ENVIRONMENT

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ABSTRACT

This document reports the process and results of the Intellectual Output IO6: GUIDELINES AND RECOMMENDATIONS FOR QUALITY ASSESSMENT IN INQUIRY BASED LEARNING ENVIRONMENT. We presented as much summative and synthesis as well as forward-looking by the assessment dimension in Inquiry based mathematics education at university level according to experience in PLATINUM Project (eight university institutions in seven different countries of the European Union). In addition, a significant portion of the report includes a Guidelines implemented in different countries and a cross-case study among example from four universities in four countries.

Keywords

University Mathematics Education, Assessment in mathematics, Inquiry based learning, Inquiry based teaching.

CONTENTS

1. Introduction

The aim of this intellectual output was to provide guidelines and recommendations for the evaluation of Inquiry Based Learning (IBL) activities in university mathematics education. To support the multi-faceted nature of inquiry-based learning exhibited in IO1 and presented in a variety of forms in other IOs, we critically assessed available measurement and evaluation tools and criteria and adopt those giving a deeper insight into IBME (see PLATINUM Book, Gómez-Chacón, Hochmuth, Jaworski, et. al, 2021).

We assume that local aims and institutional conditions for IBL activities can vary significantly, however some advice on the experience of national teams participating in PLATINUM can be shared. The contexts under which the research and evaluation tools were developed allowed us to facilitate the transfer of knowledge to other colleagues from other institutions interested in building up and fostering the progress and implementation of IBL in higher education.

The Inquiry Based Mathematics Education (IBME) took place at several different interrelated layers (Fig. 1) (Jaworski, Gómez-Chacón & Hochmuth et. al, 2021). In section three (Guideline and case study) and section four (Cross-case study (Gómez-Chacón, Brouwer Iannone & Kralova, 2021) we offered understanding and insight in the different interrelations of IBL process from different perspectives was provided.

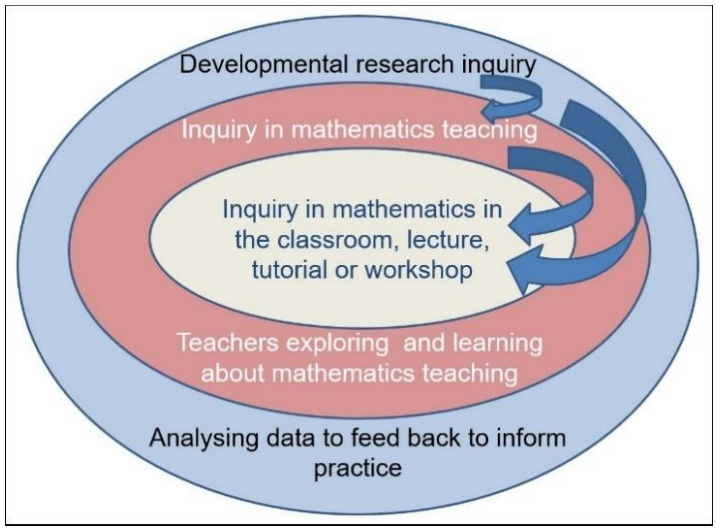


Figure 1. Interrelated layers

The reflection in this final report highlights evaluation and measurement tools, and criteria that can provide support to lecturers and stimulate their self-reflections into their own teaching, contribute to deeper understanding of the possibilities offered by the IBL and facilitate the use of inquiry-oriented learning environment.

The Leading Organisation UNIVERSIDAD COMPLUTENSE DE MADRID (UCM) with de participating Organisations, UNIVERSITETET I AGDER (UiA), UNIVERSITEIT VAN AMSTERDAM (UvA), MASARYKOVA UNIVERZITA (MUNI), GOTTFRIED WILHELM LEIBNIZ UNIVERSITAET HANNOVER (LUH), LOUGHBOROUGH UNIVERSITY (LU) developed this intellectual output.

To deliver this Intellectual Output IO6, a D-Team was set up consisting of four members belonging to some of the universities contributing to this Intellectual Output. The members belonging this D-Team are Professor Inés M. Gómez-Chacón (UCM), Dr. Natasa Brouwer (UvA), Dr. Paola Iannone (LU), and Professor Maria Králová (MUNI).

The constitution of the D-Team was effective in furthering the objectives of the IO6 and in driving forward the stages developed. It was useful to discuss points, to improve and review issues and to make decision in the proposal for the whole PLATINUM group in areas of development of IO6 (evaluating teaching, monitoring students' engagement, monitoring professional development courses and workshops).

The following steps were suggested to develop the proposal:

1. Step 1: Identification of criteria and tools that each partner is developing.
2. Step 2: Assessment Detailed guideline, application in each context for a case.
3. Step 3: Cross-Case Analysis
4. Step 4: Final report.

The report is structured as follows: first we discuss the identification of criteria and tools by the partners, then the assessment detailed guideline, the implemented in three contexts (Czech Republic, Spain and Ukraine), a Cross-Case Analysis, where we discuss in detail representative cases in the PLATINUM project, and a final section where some challenges for future research are addressed.

2. Identification of criteria and tools by the partners

This item was developed over a year and a half after the beginning of the project in September 2018. The key has been the joint work developed in the different meetings and specific workshops by the project members. The following are highlighted:

PLATINUM Project Management Meeting in Kristiansand at UiA (Norway), September 25, 2018, under the title Quality assessment in IBL environment the Professor Inés Gómez-Chacón (UCM), Dr. Andrián Riesgo (UCM) and Johanna Ruge (LUH) presented the objectives of the IO6 and the work plan. Examples of possible assessment tools were also presented as an example.

PLATINUM Project Management Meeting and Workshop in Madrid at UCM university (Spain), February 23, 2019 (UCM) initial discussion orchestrated of the IO6 by Professor Inés Gómez-Chacón (UCM) and Professor Maria Králová (MUNI). This session also distributed tasks and encouraged the creation of instruments.

PLATINUM Brno workshop June 19, 2019, Brno at MASARYK UNIVERSITY(MUNI) (Czech Republic). Although the focus of the work of IO6 at this meeting was on the presentation of the Guide by COMPLUTENSE UNIVERSITY (UCM) members (see section 3 of this report), one of the tools developed by the GOTTFRIED WILHELM LEIBNIZ UNIVERSITAET HANNOVER (LUH) team to establish connections between IO2 (Learning about Teaching: Case studies for dissemination of community of inquiry developmental practices) and IO6 was also presented. The tool chosen was "spiderweb charts". A short introduction of how we work with the spiderweb charts was given by Johanna Ruge (LUH). A group work dynamic was carried out for the application of the tool "spiderweb charts". In small groups, each partner presented the filled-out charts and explained why they choose to tick the boxes. Groups discussed the following questions: Are there different ideas about the characteristics and quality nuances? What characteristics and quality nuances fit well to the PLATINUM project? What characteristics and quality nuances need to be reformulated (please make suggestions)?

Each partner has developed various instruments or tools for evaluation according to the type of research activities. Given the variety of contexts and aims the activities

carried out by each partner will be diverse. We take this variety as a strength of the project as it provides a very beneficial spread of perspectives. The aim of this step was to create and systematically collect evidence which is included in this final report. It contributed to the identification of criteria and tools which can be used for:

a) evaluating teaching, design or innovation with respect to the provision for student engagement and conceptual learning of mathematics;

b) monitoring students' engagement (acting/learning) in inquiry-based learning activities;

c) monitoring professional development courses and workshops designed for introducing new and experienced lecturers to inquiry-based practices;

d) developing marking criteria and marking schemes for assessing students' mathematical competence in courses that include inquiry-oriented teaching-learning modules.

All information relative to the first step was shared among the PLATINUM project partners in the OwnCloud in a folder. Each partner could include all the instruments designed and used to evaluate their activities in the first year. Each partner wrote a summary of the types of activities they wanted to evaluate, what are the aspects of the activities they want to evaluate and the tools they are planning to use for the evaluation.

Following we included a summary description of criteria and tool according to contribution of some partners.

2.1. Criteria and tools by UNIVERSITEIT VAN AMSTERDAM (UvA)

At the University of Amsterdam (UvA) the report was presented by Natasa Brouwer and André Heck. In this university the inquiry-based mathematics education (IBME) was implemented in the mathematics courses in the bachelor in several natural sciences programmes other than mathematics. In these courses asynchronous online learning in SOWISO often takes place for individual learning activities next to face-to-face learning activities in the classroom. SOWISO is an online environment specifically designed for learning, practicing and assessing mathematics, statistics and other exact sciences oriented courses. During the face-to-face activities of the course in the classroom the

students work on the inquiry-based assignments to get deeper conceptual understanding of mathematics. The students work in small groups on problems and discuss them following an inquiry based- learning approach. Students used computers when working on authentic realistic problems (see UvA cases in the PLATINUM book).

The UvA PLATINUM team evaluates IBME activities at three levels: student level, lecturer level and level of lecturer professional development. Different instruments are used in this process. Some of the instruments are new and are developed within the PLATINUM project, some instruments were used already before for the evaluation but are adapted for IBME purposes and some instruments that are used for other evaluation and reflection are used for the purpose of IBME in unchanged format. In this step only the instruments and approaches were described, the results of the evaluation are not presented or discussed here (see chapters in the PLATINUM Book).

2.1.1 Instruments for students

For the evaluation of student's perception about IBME an online questionnaire was used. At the beginning of the course personal data about the students were collected. At the end of the course the students were asked to reflect on learning mathematics. Students answered questions about their background, math anxiety, test anxiety, and motivation & engagement, and they took a diagnostic mathematics test. The questionnaire was implemented in the SOWISO platform. Parts of this evaluation questionnaire were standard instruments and parts had been used before. The questionnaire in SOWISO for evaluating learning of mathematics was already used for years in several mathematics courses that use SOWISO at the UvA. Standard question sets translated into Dutch were used to measure math anxiety [Hopko, D. R., Mahadevan, R., Bare, R. L., & Hunt, M. K. (2003). The abbreviated math anxiety scale (AMAS). Assessment, 10(2), 178-182], test anxiety [Spielberger, C. (1980). The Test Anxiety Inventory. Palo Alto, CA: Consulting Psychologist Press], and motivation & engagement [Martin, A.J. (2007). Examining a multidimensional model of student motivation and engagement using a construct validation approach. British Journal of Educational Psychology, 77, 413-440]. To this student evaluation instrument, the following three questions were added that were specifically oriented on IBME:

(1) In some mathematics tasks you had to find out/discover things by yourself. Such an approach of "inquiry-based learning" has an appeal for me.

(2) A small inquiry task to be carried out in pairs for example as a bonus task. It seems to me a useful extension of the course.

(3) In lectures there were sometimes short tasks embedded (for example, inventing a method for numerical differentiation and practicing with line element fields and direction fields). I learnt much from these tasks.

2.1.2 Instruments for lecturers

The lecturers at the University of Amsterdam who take part of the PLATINUM project have established an IBME community of inquiry that was connected to one large teaching course of mathematics for Biomedical Sciences where they all have been involved as lecturers. Reflective discussions according to the IBME framework, structured oral evaluations and written narratives were used as instruments for reflection and evaluation in on this level.

These instruments were used during weekly course lecturers team meetings. The meetings were organized face-to-face. All meetings were recorded on audio and of each meeting minutes were also taken by the principal lecturer. The members of the team wrote narratives / reflections. This material collected during the whole course was used as instrument to evaluate IBME on the level of lecturer.

2.1.3 Instruments in professional development of lecturers

The University Teaching Qualification (UTQ) programme at the Faculty of Science, University of Amsterdam is organized around five lecturer's competences within the national framework for professional development of university lecturers in the Netherlands: design of teaching, provision of teaching, support and supervision of individual students, organizing teaching and professional attitude. In this UTQ programme all lecturers must observe one lecture/session that is also recorded on the video. They need to observe a lecture/session of a peer, give feedback to him/her and watch the video of the own lecture/session and reflect on teaching, behavior of students during the lecture and anticipation of the lecturer on the activity of students and evaluate this.

5 different instruments are used in this peer-evaluation and reflection process (added in attachment): (1) lecture plan having a screen play format to plan teaching, learning and (formative) assessment activities in the time frame of the lecture/session, (2) video recording of the lecture/session taken with two cameras or one 360 degrees camera, (3) peer- feedback/observation report of the observer of the lecture/session (in principle in the lecture room), (4) student evaluation and (5) reflection report of the lecturer (on the video) based on self- observation, peer-observation report and the students evaluation.

This set of five instruments is used for the evolution/reflection of the lecturers who apply IBME in their teaching in unchanged way. All lecturers who follow the UTQ programme are encouraged to design learning activities in which students are active, use critical thinking and conceptual learning that leads to deep learning. The lecturers who taught mathematics were stimulated to design IBME assignments and to introduce them in the lecture that was recorded on the video in the context of their UTQ trajectory. They were invited to join the meetings organized at the faculty about IBME or PLATINUM project.

The material of the lecturers who have introduced IBME assignments is used as instrument to evaluate IBME on this level.

2.2 Criteria and tools by Universidad Complutense de Madrid (UCM)

At the Universidad Complutense de Madrid (UCM) the report was presented by Inés Gómez-Chacón and Adrián Riesco and participating in the preparation David Gómez y Teresa Luque. In this university the IBME was implemented in the mathematics courses in the Bachelor of Mathematics, Bachelor of Mathematics and Engineering, Bachelor of Mathematics and Statistic, Bachelor of Computer engineering and Programmes of Professional Development for mathematics lecturers. The evaluation of IBME activities took place on three levels: student level, lecturer level and level of lecturer professional development. Different instruments were used in this process. The elaborated materials could contribute to the identification of criteria and tools which can be used for: evaluating teaching design, monitoring students' engagement (acting/learning) in

inquiry based learning activities, monitoring students' engagement (acting/learning) in inquiry-based learning activities

Some examples are briefly described below.

2.2.1 Instruments for monitoring students' engagement (acting/learning) in inquiry-based learning activities

The following is an example of such instruments and criteria applied.

The host course: Elements of Ordinary Differential Equations

- Introductory course in ODEs (basic theory, solution techniques...)
- 2nd year all degrees in Math (Math, Math Eng, Math & Stats)

The host problem: Persecution curve

- A hare (or mouse) describes a trajectory (typically constant speed)
- A dog (or cat) chases, always facing in the hare (typically constant speed)
- The problem can be written as ODE

Presentation of Task:

Guided inquiry: The Escape Room approach

- Since the problem is not trivial, we chose guided inquiry
- We disguised the guided inquiry as clues in a game

Aims and expectations

- The clues would resolve possible blocks from the students
- Provided with the clues, each step is a simple application of Analysis techniques
- We want to detect whether the students "grasp" the concepts
- We expected the students to finish the activity

Task

2.1. **Task 1. A very simple case.** Assume that the hare starts at the origin and runs along the y axis at a constant velocity v_l, and the dog departs from $(0, -y_0)$ and runs with constant speed v_p.

What relation between y_0, v_l and v_p guaranties that the dog will not reach the hare?

Solution. This is a simple problem of uniform motion, and the condition is simply that $v_p \leq v_l$. With y_0 one can determine the time.

2.2. **Task 2. A reasonable problem of ODE's.** Assume now that the hare departs from the origin and moves along the positive y axis with constant velocity v_l and the dog departs from $(x_0, 0)$ with constant velocity v_p.

Again, we want to find the relation between x_0, v_p and v_l that guaranties that the dog will catch the hare and, if possible, to determine the trajectory.

The lecturers' work: The clues

Students work in groups to develop the tasks. Different clues are given by the teacher as the students are blocked in the resolution of the task. They allow for progress in the inquiry process.

(1) Draw the situation.
(2) Write coordinates for the hare's trajectory.
(3) Model, from the drawing, an ODE for the situation.
(4) Consider that $y = y(x)$. Write an equation for $\frac{dy}{dx}(x)$
(5) In order to obtain this equation, take into account that the chain rule gives: $\frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dt}$.
(6) Taking derivatives with respect to x in the previous equation, deduce that

$$x\frac{d^2y}{dx^2} = -v_l\frac{dt}{dx}.$$

(7) Consider the arc-length s, which satisfies the relation

$$ds^2 = dx^2 + dy^2,$$

(8) This means that, for every change of variable α $s = s(\alpha), x = x(\alpha), y = y(\alpha)$

$$\left(\frac{ds}{d\alpha}\right)^2 = \left(\frac{dx}{d\alpha}\right)^2 + \left(\frac{dy}{d\alpha}\right)^2$$

(9) Use $\alpha = t, x$ to compute $(dt/dx)^2$
(10) Use the fact that x is decreasing with respect to t.

(11) Show that the differential equation for y can be written

$$xy'' = k\sqrt{1+(y')^2},$$

where $k = v_p/v_l$.

(12) Through a substitution $p = y'$ simply the equation above to a first order equation in p.

(13) Take into account that $p(x_0) = dy/dx(x_0) = 0$

(14) Use the change in variable $x = \sinh\tau$

(15) The solution is

$$p(x) = \sinh\log\left(\frac{x}{x_0}\right)^k$$

(16) Use the definition of $\sinh\tau$

(17) One can rewrite p as

$$p(x) = \frac{1}{2}\left(\left(\frac{x}{x_0}\right)^k - \left(\frac{x}{x_0}\right)^{-k}\right)$$

(18) Integrate to recover $y(x)$.

$$y(x) = C + \begin{cases} \frac{1}{2}\left(\frac{1}{k+1}\left(\frac{x}{x_0}\right)^{k+1} - \frac{1}{1-k}\left(\frac{x}{x_0}\right)^{1-k}\right) & k \neq 1 \\ \frac{1}{2}\left(\frac{1}{2}\left(\frac{x}{x_0}\right) - \log\left(\frac{x}{x_0}\right)\right) & k = 1 \end{cases}$$

(19) Compute C from the initial data.

(20) The dog does not reach the hare if $y(0) = +\infty$ (this corresponds to a vertical asymptote).

(21) The no reach condition is $v_p \leq v_l$.

Experimental set-up and evaluation instruments

The "Escape Room" task was carried out over three sessions. Two sessions where students solved the task in collaborative groups. And the third session, coordinated by a teacher from the PLATINUM team, focused on the discussion of the common aspects of the solutions contributed by the groups (mathematics, dimension of the interactions, levels of enquiry, etc.).

Students were split into 4 groups of 5-6 people. The first two sessions were recorded by group, with students' laptops. We use VLC to record the webcam. This allowed lecturers to observe students' activities, paying special attention to the interactions between the students in the processes of inquiry. The third session was recorded with two cameras to capture a panoramic view of the class group. (see Figure 2).

Figure 2. Classroom on discussing common aspects

In these sessions, students were given the problem and asked to describe their approaches to solving the problem on a protocol including steps in the resolution, explanations of the difficulties they might face, and strategies they would use in order to solve it. The third session was used for working on the Task and discussing common approaches to the process and the difficulties that arose (based on questionnaire answer). The video recording helped us to observe students' activities, paying special attention to the interactions between the students and the lecturers.

Questionnaire (anonymous survey)

1. How was the experience of facing an open problem, for which you are not prescribed an approach (in principle)?
2. What difficulty do you have when facing a word problem instead of one in mathematical formulation?
3. When you were stuck, what caused it?

was the use of clues useful?

4. After the activity, how would you grade your domain of analysis techniques?

do you think you chose well which to use?

5. What do you think about this way of posing problems?

Do you think it would positive to add more activities of this nature to this course?

In which other subjects would you use it?

Can you think of any activity?

6. Rate this activity from 1-10, where 10 is the maximum

In summary: Formative assessment

1. Evidence to be included and how to evaluate of students

Students' interactions during the activity are recorded in video, and protocols of resolution, their rough work is collected.

Lecturers (teachers) interact with the students for questions and monitoring during the activity. Final questionnaire and group interview.

2. Resolution of activities. Final resolution of the problem (protocol of the group). It allows to evaluate

- **Activity planning and implementation (content and process)**: From the complete process and it is collected in session 3.
- **Content knowledge:** test tools (students' assessment) to measure knowledge competences): from the complete process, the questionary for self-reflection and the protocol of problem resolution. All of these are collected in session 3.
- **Procedural knowledge** (non-test tools to measure attitude and skill competences): from the complete process and it is collected in stage.

3. **Lecturers (Teacher)-student interviews**.

4. **Videos: working group and whole group discussion.**

2.2.2 Monitoring professional development courses and workshops designed for introducing new and experienced lecturers to inquiry-based practices

In what follows we describe some examples that the UCM team used. For more information, see sections 2 and 3 of the report.

2.2.2.1. Novice Lecturer- Initial questionnaire in the courses "Motivation, expectation about the course and the problems teachers face"

Reference: Gómez-Chacón (2017). Actividad formativa de doctorado en docencia Universitaria. Doctorado IMEO and Doctorado en Investigación Matemática, Facultad de Ciencias Matemáticas, UCM. And UCM Innovation Project 214, UCM:

Introduction: Initial questionnaire for the courses. The Aims are the following:

- Helping lecturers address and formulate teaching challenges.
- Identifying questions, based on a study and research path for teacher education, an inquiry-based teaching format focused on the study of a professional teaching question.
- Connecting to the knowledge to be taught.

Type: Online Questionnaire

Item categories: Qualitative**.** Some Items:

Send 2-3 issues (problems, difficulties, or doubts) you find or think that may appear when teaching at University level. Use as much space as you need for each one, describing them as faithfully as possible.

Issue 1: Problem, difficulty, or doubt

Issue 2: Problem, difficulty, or doubt

Issue 3: Problem, difficulty, or doubt

Implementation:

Implementation in PLATINUM 2019: 36 participants

Implementation in PLATINUM 2021: 45 participants

2.2.2.2. Teaching practice in inquiry-based education in mathematics-Survey

Reference: Gómez-Chacón, I. & Luque, T. (2019) Teaching practice Fonction and Rolle Theorem by Novice Lecturer. UCM. Nov. 2019 Computer Faculty. Engineering Students.

Gómez-Chacón (2019) Novice lecturer. Developing teaching intuition: the mediation of cognition and action at the individual and social levels. UCM

Introduction: To explore some pedagogical and didactic implications in the teaching practice for developing the capacity to learn to teach through the inquiry approach and in an inquiry community (at individual level, mediation and social level, attitudinal and engagement dimensions). To go deepen how advanced mathematical concepts are characterized by complex interactions between intuitive and rigorous reasoning processes

Type of instrument: Paper and pencil questionnaire

Item categories: Qualitative and quantitative

Items:

Examples of items were included in questionnaire Following:

1. Did the **design of the tasks** give you a better understanding of the situation of your students with respect to some concepts?

a. Yes, they did…

b. No, they don't since…

2. During **the design of the tasks,** what did you find easier? And more difficult? Why?

a. Easier part:

b. More difficult part:

3. During **the development of the tasks** in the classroom with the students, what did you find easier? And more difficult? Why?

a. Easier part:

b. More difficult part:

4. When the class session ended, students seemed to have significantly changed their metacognitive behaviour and mathematical attitudes. In particular:

	1	2	3	4	5	0
They make connections between the various topics;						
They work in an inquiry way						
They study in a critical way;						
They identify their own doubts;						
When faced with a written test, they activate control strategies;						
They appear to be more interested in the subject;						
Most of all, they feel more effective, partly because they realize why they failed the understanding in the past						
Others (please formulate them)						

5. In my opinion the more specific components responsible for the success are:

The homogeneity of the group, that allows students to accept their own difficulties, and to work in a relaxed and collaborative way;	1	2	3	4	5	0
The homogeneity of the group, that guarantees common objectives.						

Students' trust in the teacher. This trust is reinforced by a continuous feedback: effort →success→ effort →success …						
The organization of the didactic material. This is organized in order not to allow students to face tasks without having the necessary knowledge, and to point out the difference in processes (not only at the mental level but also at the affective one) involved in *exercises* and *problems*.						
Others (please formulate them)						

2.3. Criteria and tools by MASARYKOVA UNIVERZITA (MUNI)

At the MASARYKOVA UNIVERZITA (MUNI), the report was presented by Professor Maria Králová and Lukáš Másilko. In this university IBME was implemented in the mathematics courses. The criteria and instruments are as follows.

2.3.1 Instruments for students and design innovation

Two dimensions were assessed:

- Evaluating teaching, design or innovation concerning the provision for student engagement and conceptual learning of mathematics.
- Monitoring students' engagement (acting/learning) in inquiry-based learning activities.

IB tasks (within a framework of Mathematics and Statistics courses) were supplemented with the observer's evaluation. The evaluation assessed what planned aims were covered by the IB tasks. Two aims at least were considered: 1) Was the intended knowledge achieved by students? 2) Were students active and participated actively in the learning process? X) We appreciate further suggestions.

To monitor students' engagement in inquiry-based learning activities immediately after the seminar/lecture with IB tasks students will receive a survey with questions related to the benefit of the IB task from their perspective. We don't have the questions yet – any suggestions from the audience?

Benefit of some IB tasks in Mathematics at the Faculty of Education was evaluated via worksheets which will be provided to hopefully both experimental and control group of students.

Also, it was recorded of "Community of Inquiry" meetings.

2.3.2 Developing grading criteria for assessing inquiry-oriented teaching-learning modules

The MIUNI team would like to notice that didn't have teaching modules completely IBME. It was developing grading criteria for assessing students' mathematical competence solving tasks, not for a whole course or subject. It was not able to grade separately competences achieved solely via IB tasks. For that reason, we added additional points to those students who actively participate in developing ideas inspired by IB tasks. These points were added to test scores and can improve the final grades providing they passed the final test.

An example of a questionnaire used was an anonymous survey). The items in the questionnaire were as follows:

1. How was the experience of facing an open problem, for which you are not prescribed an approach (in principle)?

2. What difficulty do you have when facing a word problem instead of one in mathematical formulation?

3. When you were stuck, what caused it? was the use of clues useful?

4. After the activity, how would you grade your domain of analysis techniques? do you think you chose well which to use?

5. What do you think about this way of posing problems? Do you think it would positive to add more activities of this nature to this course? In which other subjects would you use it? Can you think of any activity?

6. Rate this activity from 1-10, where 10 is the maximum

what worked for whom in which context.

2.4 Criteria and tools by Brno University of Technology (BUT)

At the Brno University of Technology (BUT) by Josef Rebenda and Zuzana Pátiková. For BUT Community of Inquiry, now of step 1, we considered tools and instruments which can be used for: evaluating teaching, monitoring students engaegement, developing grading criteria and marking schemes for assessing students' mathematical competence.

Description of tools and instruments used:

a) teacher reflection/narratives, discussion with and feedback from observing colleagues, surveys/questionnaires (paper/online), including written feedback from students, written or oral feedback from students upon request, students' assessment in tests and exams.

b) teacher observations, observing colleagues 'notes, surveys/questionnaires (paper/online), including written feedback from students.

c) accompaniment of the individual projects, homework, written exam, oral exam, etc.

It is important to note that the used the tools and instruments listed in part b) are also for monitoring students' engagement (acting/learning) in all learning activities, not only those which are inquiry-based (See case study in the chapter 17 of PLATINUM Book, Rebenda, J. Patrova, Rebendová, Sedlalová, & Demchenko, (2021)).

2.4.1 Example of questionnaire and experience

Questionnaire – Definite Integral

1. What do I understand/imagine under the notion/symbol of the "definite integral" $\int_a^b f(x)dx$? What do I expect/imagine that I am able to calculate using the "definite integral"?

2. How would I proceed to find/determine area of the shape inside the curve in the picture? (Figure 3)

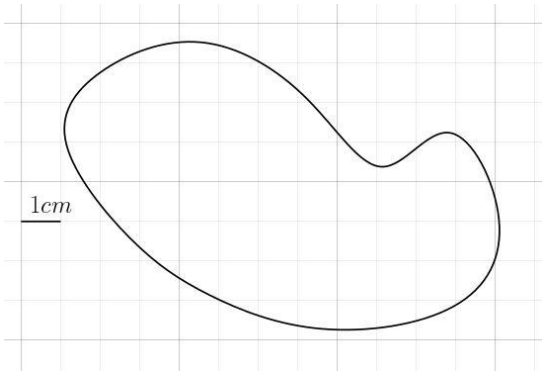


Figure 3

3. What comes to my mind when I see/hear the term "upper/lower sum"?

4. In some classes/seminars we worked with touchscreen devices. Rate the level of work with tablets and electronic materials.

☐ Very easy ☐ Easy ☐ Normal ☐ Difficult ☐ Very difficult

5. What would help me understand/learn math better?

Questionnaire - experience

Total number of submitted questionnaires: 44

Time period of submission: May - June 2020

Question 1:

41 meaningful (expected) answers (93%)

Question 2 (some answers counted in multiple categories):

Strategy of summing up squares or similar: 32 (73%)

Use of the (definite) integration: 13 (30%)

Another approach: 2 (5%)

No answer: 4 (9%)

Question 3:

Correct or close to correct answer: 5 (11%)

None or wrong answer: 39 (89%)

Question 4 (percentage is calculated from provided answers):

Choice 1 (Very easy): 19 (46%)

Choice 2 (Easy): 15 (37%)

Choice 3 (Normal): 6 (15%)

Choice 4 (Difficult): 1 (2%)

Choice 5 (Very difficult): 0

No answer: 3

Question 5:

Answers to Question 5 have not been relevant to the context of the teaching unit.

Reflection: More than 90% of students have successfully formed an idea of the concept and symbol of a definite integral and what can be calculated using it. About one third of students can combine the idea of definite integral with a practical task. But most would still prefer a less sophisticated method/procedure to calculate/estimate the area of a general shape. According to the results of question 3, students (almost 90%) were unable to understand the underlying concept on which a definite integral is based or did not absorb/recall the terminology used. Next time, we should figure out how to complement existing activities to help the students understand and absorb the concept and notions better/deeper. Concerning the work with tablets, most students can handle digital technology and such activities can be included in teaching design.

Since students were asked to fill in the questionnaire 2-3 months after the lessons/seminars, the results show only long-lasting knowledge. Almost 10 percent of the students did not understand the concepts introduced in the particular seminar. Most of those 10 percent then have not been successful in the final exam. Some of those 10 percent also did not finish the preceding Mathematics 1 (=Calculus 1) course successfully.

2.5 Criteria and tools by GOTTFRIED WILHELM LEIBNIZ UNIVERSITAET HANNOVER (LUH)

At the GOTTFRIED WILHELM LEIBNIZ UNIVERSITAET HANNOVER (LUH) this item was presented by Johanna Ruge, Reinhard Hochmuth, Jana Peters and Sara Khellaf. In this

university the IBME activities were implemented in the mathematics courses. The criteria and instruments were based in formative evaluation. The type of activities is the following:

2.5.1 Evaluating teaching, design or innovation with respect to the provision for student engagement and conceptual learning of mathematics

Design and teaching

- ***Spider charts***

  - Teaching- learning situation for which a new design shall be developed, can be analysed and potential spaces for inquiry identified
  - Potential for inquiry of current design can be analysed
  - Formative evaluation is necessary to assess, if context factors are expected to hinder inquiry in the given situation. This can inform the design of activities, e.g. through the Identification of central dimensions of the tasks
  - Basis for monitoring the quality of implementation - Starting point for further qualitative (e.g. categories for questionnaires) and quantitative (e.g. manifest or latent factors in cause-effect relationships) evaluation steps
  - Easy applicable reflection tool (See Figure 4, 5 and 6 and more information in Khellaf, Hochmuth, Peters, Ruge (2021)

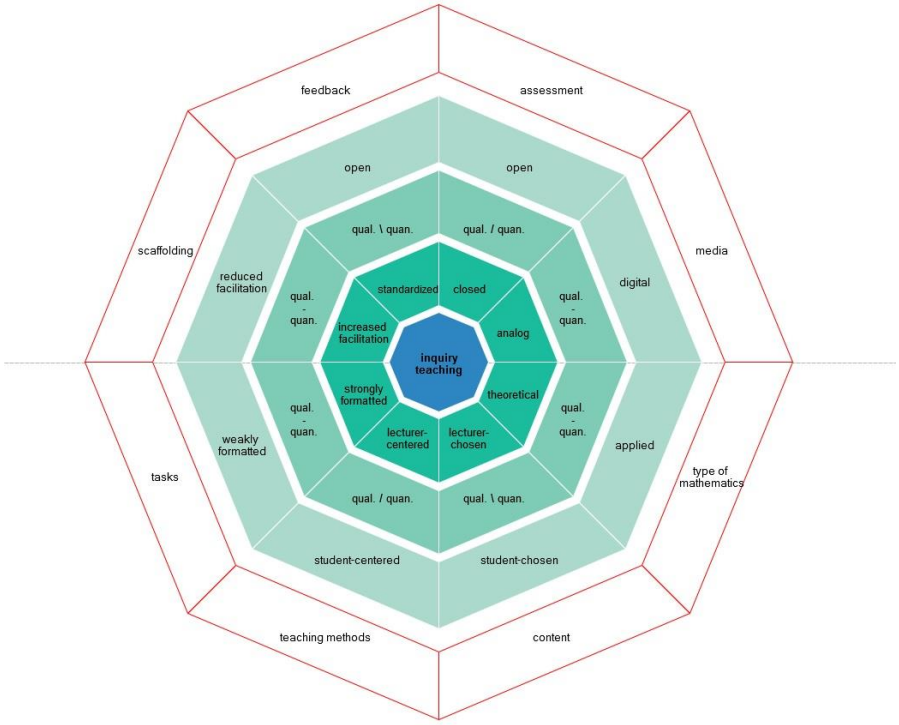


Figure 4: Evaluation Instruments spider chart

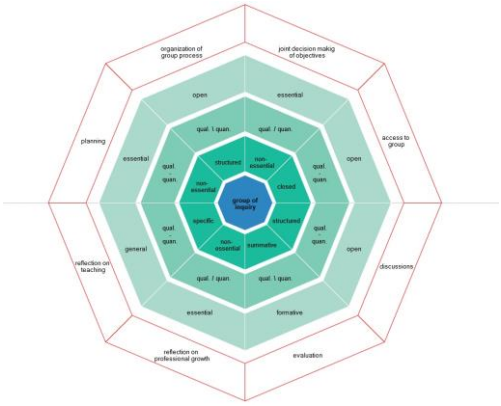
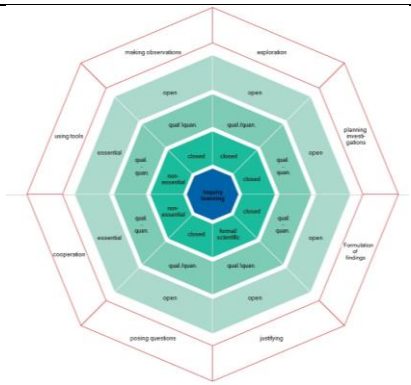
	

Figure 5. Evaluation Instruments spider chart group of inquiry	Figure 6. Evaluation Instruments spider chart inquiry learning

- ***Table for task analysis***

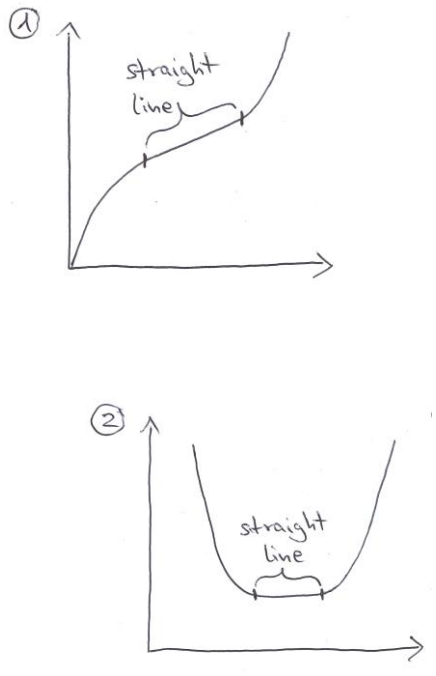
  - Tasks can be analysed

- Potential for inquiry can be reflected and, partly, identified
- Classification of tasks
- Transfer of tasks Subject matter to be taught (See Figure 7).

Subject matter to be taught

- ***ATD-Analysis***

- Praxeological analysis to reconstruct typical/ideal praxis and logos-block. The table for task analysis (see above) can support this.
- Typical/ideal discourses can be reconstructed, which might feed into question generation in design by showing "blind spots" in common curricula
- Creating questions for inquiry

	An upper high school class is revisiting inflection points in mathematics. One of the pupils draws onto the sheet of his neighbor graph 1 and comments: Listen, I always wondered if a function that looks like this has inflection points everywhere on the straight line. What do you think? The neighbor, visibly amused, adds graph 2 below and replies: Look, can't you do the same with a parabola? If you flatten it on the lower side like this, wouldn't it then have lots of extremums as well? Infinitely many, even!
Figure 7. Example 1: task introduction with teacher students	

Goal dimension / Task format	Task aims at learning of **inquiry-strategies**	Task promotes **discourse/reasoning about techniques**	Task aims at **inner-mathematical cross-linking of knowledge**	Task aims at **interdisciplinary cross-linking of knowledge**
Closed				
Open with respect to solution path				
Open with respect to solution / result				
Open with respect to knowledge used				

Figure 8: Chart, overview of task characteristics

2.5.2 Monitoring students' engagement (acting/learning) in inquiry-based learning activities

Students' engagement (acting/learning) in inquiry-based learning activities

- ***Qualitative document analysis incl. ATD***
  - Results of students' engagement: Documents that students produced (texts, learning materials, etc.)
  - Describing and analysing the student products

- ***Qualitative text/transcript analysis incl. ATD***
  - Results of students' engagement and live engagement in specific sessions:
  - Audio from sessions
  - Describing and analysing format-specific student products

- ***Qualitative text/transcript analysis incl. ATD***
  - Self-reported engagement with and opinion on course materials / specific course task:
  - Interview material
  - Describing and analysing format-specific student products

Students' overall engagement (acting/learning) in course as a whole

- ***Questionnaire about course***

- Student's opinions on course and self-reported opinions on their own activities in the course
- Easy to implement; can give some general idea of certain elements of the course and ideas for possible improvements.

2.5.3 Monitoring professional development courses and workshops designed for introducing new and experienced lecturers to inquiry-based practices

Professional development courses

- Spider charts [See above 2.5.1)]
- Table for task analysis [See above 2.5.1)]
- ATD-Analysis [See above 2.5.1]

The evaluation tools we describe in a) were designed within the context of mathematics teacher education course, so it is also applicable for professional development

2.5.4 Developing grading criteria and marking schemes for assessing students' mathematical competence in courses that include inquiry-oriented teaching-learning modules

Assessing student's mathematical competence in courses that include inquiry-oriented teaching- learning modules

- ***Term paper (open format)***

- Contents of a text, which was created in an inquiry process (by definition) and the mathematics in this text. (See Figure 9).
- Describing and analysing the student products
- Assessment of inquiry tasks must leave room to show the results of the inquiry activity that are connected to the type of inquiry expected.

Assessing student's mathematical competence in courses that include inquiry-oriented teaching- learning modules

- ***Results of past multiple-choice exams.***

  - The knowledge obtained in a course which featured inquiry elements.
  - Assessment of the acquired knowledge, no matter whether it was obtained through inquiry or other means (e.g. rote-learning)

Assessment category	**Features of the text which are relevant for this category**
Style / expression / language (20%)	Does the text adhere to grammatical rules? Does the text adhere to rules of spelling and puctuation?
	Is the language of the text appropriate for an academic context? a) expression, word choice b) precision
Citation (10%)	Does the text adhere to common rules of citation? a) clear marking of quotes b) comprehensible and coherent reference format c) cited work can be found with the given reference
Method (20%)	Is the context of the text made clear? a) clear topic / question b) relevant / necessary material can be found c) references to material are intelligible
	Does the treatment of the topic draw in an appropriate manner on prior knowledge and material provided by the lecturer?
Content (50%)	Does the text contain the definitions of important technical terms? Are technical terms used correctly? (mathematical and didactical)
	Are explanations of important theories / positions / arguments from the literature factually correct? (Mathematics and Didactics)
	Quality of reasoning: Are statements well justified or are there open questions?
	Quality of reasoning: Is the line of reasoning clearly communicated and structured?
	Does the text provide an answer to the initial question?
Creativity (Bonus)	Personal contribution: Does the text concentrate on summarizing literature or does it communicate a personal standpoint or original ideas?

Figure 9. Assessment categories for term paper

2.6 Criteria and tools by LOUGHBOROUGH UNIVERSITY (LU)

At LOUGHBOROUGH UNIVERSITY (LU) this item was presented by Paola Iannone. Inspired by the principles of realistic evaluation:

> Whereas the question which was asked in traditional experimentation was, "Does this work?" or "What works?", the question asked by us in realistic evaluation is "What works for whom in what circumstances?" (Tilley, 2000, p. 4)

Case of: a group of mathematicians and mathematics educators (5 – 8) from the same institution meeting through the course of three years to discuss teaching and learning of mathematics at university level.

Evaluation of what? What worked for whom in what circumstances. In detail:

- What activities were conducive to effective reflection on practice?
- What activities had a tangible impact on practice?
- What activities were not deemed to be beneficial?
- What were the aspects that facilitated participation to the sessions?
- What were the aspects that preventing participation to the sessions?
- What were the contextual factors that facilitated (or prevented) participation to the sessions?

To answer these questions, we planned to carry out a series of semi-structured interviews with the main stakeholders:

- Participants to the session of the CoI;
- Director of studies, Head of Department;
- Colleagues who are interested in teaching and learning of mathematics (from our experience) but did not take part in the sessions of the CoI.

We planned to interview 10 colleagues and when the COVID-19 outbreak started we were in the process of drawing an interview schedule and recruit participants. Hopefully we will resume these activities soon.

Analysis. Once we will have the narratives from the interviews (e.g. transcripts), we propose to analyse them thematically to contribute to the original question: what worked for whom in which context.

Materials to share with colleagues (also outside PLATINUM): participant selection criteria, interview schedule, themes emerging from the qualitative analysis, list of contextual factors impacting on the outcome (for comparison).

2.7 Initial findings

With the synthesis expressed here we wanted to show the variety and breadth of criteria and instruments put forward by the partners in the PLATINUM consortium. The option in this first step was to allow variety and richness to emerge, and then in the following steps to detect convergences and refine the proposal in PLATINUM.

The experience of six national teams shared here helped us to describe the IBL assessment that take place at several different interrelated layers (Fig. 1). The diversity of approaches was appreciated, and rather than being considered a limitation, we believe that it could offer multiple perspectives on the subject. It paved the way for the next phases of the work, as can be seen in the Guideline document and the Cross-case study.

3. Assessment detailed guideline, application in each context for a case

This item was developed since February 2019 until the end of the project. The key aspects were introduced in the different general meetings and specific workshops. The following are highlighted:

PLATINUM Brno workshop June 19, 2019, Brno at Masaryk University (Czech Republic). At this meeting a long work on IO6 was carried out among all partners. First, Dr. M. Teresa Benavent (UCM) presented the Guide under the title Guidelines and Recommendations for Quality Assessment in IBME Environment elaborated by the UCM team. Some examples from the UCM were shown, followed by discussion on small group work about its applicability in other contexts and comments, suggestions for

improvement in assessment area and the expectation about the product at the end. The sharing was coordinated by Professor Inés Gómez-Chacón (UCM), Dr. Natasa Brouwer (UvA) and Professor Maria Králová (MUN).

Also, on another level and in connection with the IO2, related to one of the assessment instruments developed in the IO2, a group work dynamic was carried out for the application of the tool "spiderweb charts". Short introduction of how we work with the spiderweb charts was given by Johanna Ruge (LUH). In small group work, each partner should shortly present the filled-out charts and explain why they choose to tick the boxes. Groups should discuss the following questions: Are there different ideas about the characteristics and quality nuances? What characteristics and quality nuances fit well to the PLATINUM project? What characteristics and quality nuances need to be reformulated (please make suggestions)? Group leaders ensure to keep a record.

PLATINUM Project Management Meeting at Loughborough University (UK), January 23Th- 25Th, 2020. This meeting focused on assessing the Long-Term Effects of Inquiry-Based teaching. Professor Inés Gómez-Chacón (UCM) and Dr David Gómez (UCM) presented how to operationalise the Guide developed for assessment in IBME through a case study: A Case Study about Elements of Ordinary Differential Equation at the UCM. Different partners undertook to describe and present a case study using the Guide: Masaryk University and Borys Grinchenko Kyiv University.

A monographic document gathers all this information with the title "Guidelines for Evaluation of Inquiry-Based Mathematics at University Level" (see Benavent et. al., 2021, Guideline Document). This document provides a detailed methodology for designing and evaluating Inquiry-Based Mathematics Assessment in university mathematics education. The objective is twofold: on the one hand, it must be useful for designing teaching activities coherent with Inquiry-based teaching, focusing in both the evaluation with respect to the learning and engagement of students and the evaluation of the design and implementation of the task itself. On the other hand, it helps lectures to develop schemes for learning outcomes and mathematical competence, as well as to monitor and assess students' engagement in Inquiry-based activities.

This methodology is composed of three stages: before, during, and after the inquiry. These stages include different subtasks that instructors must follow to fulfil the objectives described above.

Finally, these guidelines are exemplified by a collection of experiences that illustrate how they can be used in different contexts. Lecturers can follow these examples when designing their own activities. The five cases considered five cases of studies are:

- Inquiry in Computer Science, developed at the Universidad Complutense de Madrid by Adrián Riesco. This case study presents a unit for teaching Logic to Master's students.

- Inquiry in Calculus, developed at the Universidad Complutense de Madrid by David Gómez, Teresa Luque e Inés Gómez Chacón. This case study presents how to introduce differential equations by means of an escape game.

- Inquiry in Mathematical Analysis, developed at the Borys Grinchenko Kyiv University by Mariia Astafieva Oksana Hlushak and Oksana Lytvyn,

. This case study presents an activity used to teach convergence of series.

- Inquiry in Statistics, developed at the Masaryk University by Maria Králová. This case study presents an exercise to deduce the meaning of probability by means generalization.

- Inquiry in Statistics, developed at the Masaryk University by Maria Králová. This case study describes an activity for learning how to avoid bad questions in questionnaire design.

In the document "Guidelines for Evaluation of Inquiry-Based Mathematics at University Level" we presented a brief description of the cases, and we focus here in the analysis of their assessment and report our main conclusions. Finally, it was presented and discussed about the stages in the inquiry process.

4. Cross-Case Analysis

The Cross-case study was developed during 2020 and finalised in 2021. Various general events have helped all project partners to work together. We highlight the following:

PLATINUM Project Management Meeting in Loughborough University (UK), January 23Th- 25Th, 2020. In this meeting it was focus on assessing Long-Term Effects of Inquiry-

Based teaching. Professor Inés Gómez-Chacón (UCM) and Dr David Gómez (UCM) presented how to operationalise the Guide developed for assessment in IBME through a case study: A Case Study about Elements of Ordinary Differential Equation at the UCM. Different partners were engaged to describe and present a case study using the Guide: Masaryk University,

PLATINUM Project Management Meeting in Leibniz University Hannover (Germany), May 14, 2020. It has focused on preparation and submission of deliverables – discussion led and moderated by Professor Inés Gómez-Chacón, Dr. Paola Iannone, Dr. Nataša Brouwer, Professor Maria Králová. We gave priority to Assessment Detailed guideline, application in each context for a case and Cross Case Analysis that which will be included in the chapter of the PLATINUM book.

The Chapter 9 of PLATINUM Book gathers all this information (Gómez-Chacón, Brouwer, Iannone & Králová, 2021). Based on the experience of the PLATINUM consortium, case studies will be reported and analysed through a cross-case analysis methodology (Borman et. 2006; Hopko, 2003). We will examine different evaluation and measurement tools which have been used within IBL environments in the Czech Republic, Netherlands, Spain and the United Kingdom. To support the multi-faceted nature of inquiry-based learning we will critically assess available evaluation tools and criteria and adopt those giving a deeper insight into IBL. We assume that local aims and institutional conditions for IBL activities can vary significantly therefore advice on the experience of four national teams were shared. Investigation on the contexts where the research and evaluation tools were developed allowed us to facilitate the transfer of knowledge to other colleagues from other institutions interested in building and fostering the progress and implementation of IBL in higher education. Our intention in this chapter was not to cover all possible evaluation tools exhaustively. Instead, we would like this chapter to serve as an inspiration for other communities to adjust what they find valuable in the evaluation methods described here to their setting.

5. Conclusion

This report describes the activities in the context of the IO6 of the PLATINUM project and the Guidelines and Recommendation that resulted from these activities. The results

of this intellectual output are published in a monographic document entitled "*Guidelines for Evaluation of Inquiry-Based Mathematics* at University Level" (Benavent et al., 2021, and in the Chapter 9 of the PLATINUM book: Evaluation of Inquiry-Based Mathematics Education (Gómez-Chacón, Brouwer, Iannone & Kralova, 2021).

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7. References

Benavent, M. et al. (2021) Guidelines for Evaluation of Inquiry-Based Mathematics at University Level. The PLATINUM Project. Madrid: Universidad Complutense de Madrid.

Borman, K.M, Clarke, C., Cotner, B., and Lee, R. (2006). Cross-Case Analysis, In Green, J.L., Camilli, G., Elmore, P.B., Skukauskaiti, A., Grace, E. (Eds), Handbook of complementary methods in education research. (pp. 123-140). Washington, D.C.: American Educational Research Association. Mahwah, N.J. Lawrence Erlbaum Associates.

Gómez-Chacón, I.M., Brouwer, N., Iannone, P., and Kralova, M. (2021). Evaluation of Inquiry-Based Mathematics Education, In Gómez-Chacón, I.M., Hochmuth, R., Jaworski, B., Rebenda, J., Ruge, J., Thomas. S. (Eds), Inquiry in University Mathematics Teaching and Learning: The Platinum Project. (pp. 127- 146). Brno: MUNI, Masaryk University Editor.

Gómez-Chacón, I.M., Hochmuth, R., Jaworski, B., Rebenda, J., Ruge, J., Thomas. S. (Eds) (2021) Inquiry in University Mathematics Teaching and Learning: The Platinum Project. Brno: MUNI, Masaryk University Editor. DOI: 10.5817/CZ.MUNI.M210-9983-2021.

Heck, A. (2017). Using SOWISO to realize interactive mathematical documents for learning, practising, and assessing mathematics. MSOR Connections, 15(2), 6–16.

doi.org/10.21100/msor.v15i2.412

Hopko, D. R., Mahadevan, R., Bare, R. L., & Hunt, M. K. (2003). The Abbreviated Math Anxiety Scale (AMAS). Assessment, 10(2), 178–182. doi.org/10.1177/1073191103010002008

Jaworski, B., Gómez-Chacón, I.M. & Hochmuth, R. (2021) Conceptual Foundations of the PLATINUM project, In IGómez-Chacón, I.M., Hochmuth, R., Jaworski, B., Rebenda, J., Ruge, J., Thomas. S. (Eds), Inquiry in University Mathematics Teaching and Learning: The Platinum Project. (pp. 7- 28). Brno: MUNI, Masaryk University Editor.

Khellaf, S., Hochmuth, R., Peters, J. Ruge, J. (2021) Spidercharts: A Tool for Describing and Reflecting IBME Activities. In Gómez-Chacón, I.M., Hochmuth, R., Jaworski, B., Rebenda, J., Ruge, J., Thomas. S. (Eds), Inquiry in University Mathematics Teaching and Learning: The Platinum Project. (pp. 29- 70). Brno: MUNI, Masaryk University Editor.

Martin, A.J. (2007). Examining a multidimensional model of student motivation and engagement using a construct validation approach. British Journal of Educational Psychology, 77, pp. 413-440.

Martin, A.J. (2007). Examining a multidimensional model of student motivation and engagement using a construct validation approach. British Journal of Educational Psychology, 77, 413-440.

Pawson, R. & Tilley, N. (1997). Realistic Evaluation. London: Sage.

Rebenda, J. Patrova, Z, Rebendová, G. Sedlalová, E. & Demchenko, H. (2021) Development of a Community of Inquiry Based on Reflective Teaching. In Gómez-Chacón, I.M., Hochmuth, R., Jaworski, B., Rebenda, J., Ruge, J., Thomas. S. (Eds), Inquiry in University Mathematics Teaching and Learning: The Platinum Project. (pp. 307- 326). Brno: MUNI, Masaryk University Editor.

Spielberger, C. D. (1980). The Test Anxiety Inventory. Consulting Psychologist Press.

Tilley, N. (2000). Realistic evaluation: an overview. In Founding Conference of the Danish Evaluation Society (8). Retrieved on the 04.06.2019 from http://healthimpactassessment.pbworks.com/f/Realistic+evaluation+an+overview+-+UoNT+England+-+2000.pdf.