Plotting a function

In this task we get to work with differentiation of functions in R and we will look at the difference between exact and numerical methods.

To begin, we need a sequence of t values to which we can apply our mathematical function.

Task Create a sequence of t values between 0 and 10 with increments of 0.1, and call this sequence `t.data` . Use the built-in R function `seq` .

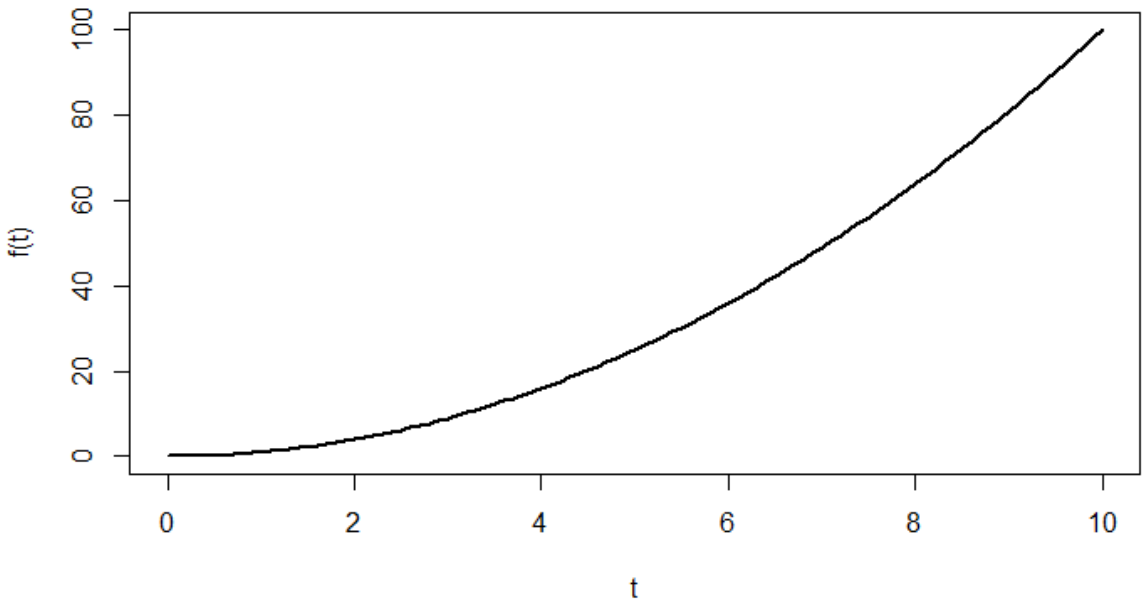
Once we have the t data we can define a function. Let's start with the function $f(t) = at^2 + b$.

Task Write a function `test.function` which has t and the two parameters a and b as arguments in input, and which returns the above function. Allow default values 1 and 0 for the parameters a and b, respectively.

Apply the function (with $a = 1$ and $b = 0$) to the sequence of t values and name the resulting sequence `test.data` . To check if everything went well, we make a scatter plot of the obtained data set.

Task Use the built-in `plot` function to create a plot of `t.data` vs. `test.data` .

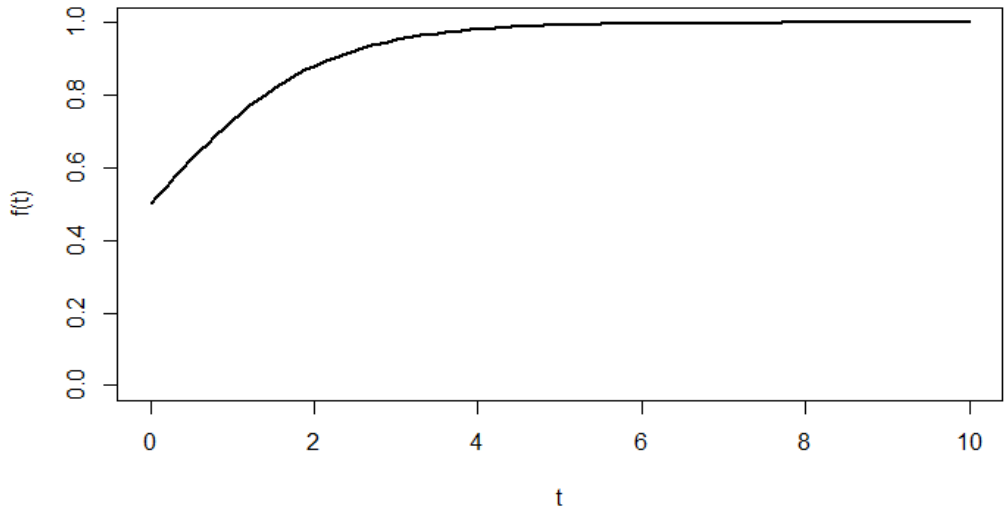
Hint: When everything goes well, the plot should look like the following diagram.



We will now consider the function $f(t) = \frac{1}{1+e^{-t}}$. This is an example of a function that models restricted growth. Can you predict what happens to function values when t becomes very large? Why is this restricted growth?

Write a new function called `function1` which again accepts t values as input and returns the function values of the above function as output. Then create a sequence of function values and name this sequence `ft`. Also construct a scatter plot in which you limit the display of y values to the range from 0 to 1 via the option `ylim`.

Hint: When everything goes well, you should see the following.



Exercises: Computing a derivative

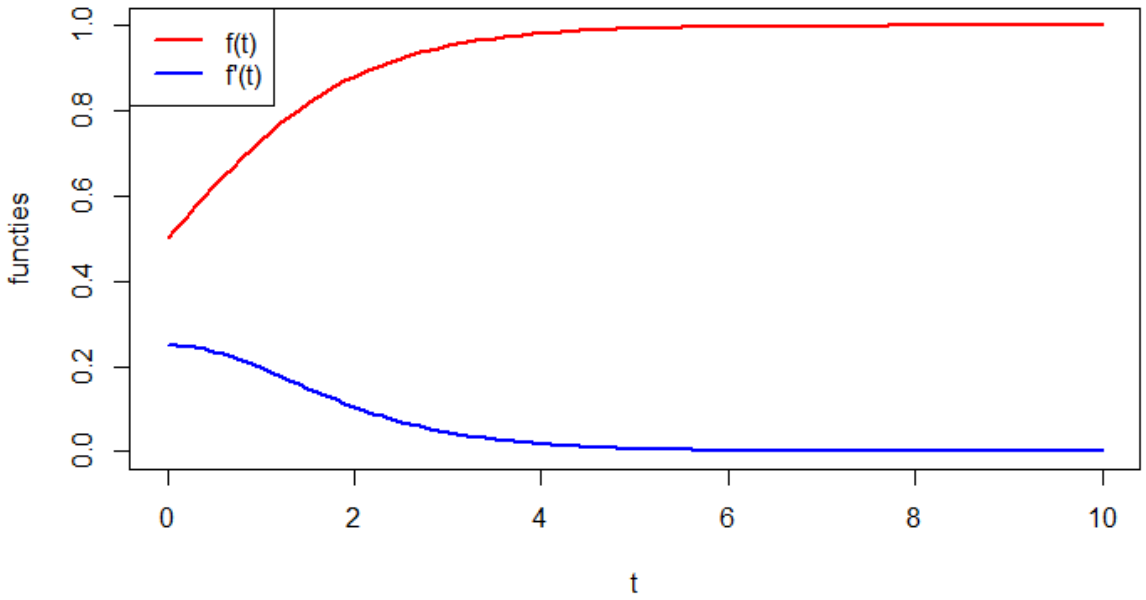
Question 1

Compute the derivative of $f(t) = \frac{1}{1+e^{-t}}$.

$f'(t) = \frac{df}{dt} =$ .....

Also add a legend that makes clear which functions are displayed.

When all goes well, the plot should resemble the following diagram.



Numerical derivatives via 2-point and 3-point difference formulas

So far, we have created a data set for the function $f(t) = \frac{1}{1+e^{-t}}$ where t runs from 0 up to and including 10 s with increments of 0.1 s. You have computed the derivative of this function, too.

However, in order to compute the derivative of an arbitrary data set on a computer, it is necessary to differentiate numerically. One approach is to use the ratio of the differences in the $f(t)$ direction and the corresponding differences in the t direction.

The numerical derivative can be determined in several ways. Here we will focus on determining numerical derivatives via the so-called *two-point forward* and *3-point difference formula*.

The 2-point forward difference formula

When for a 'decent' function $f(t)$ and a certain step size Δt the function values at t_0 and $t_0 + \Delta t$ are known, then the derivative $f'(t_0)$ be cn approximated with the **forward finite difference.** This is given by:

$$f'(t_0) \approx \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}, \quad \text{for small positive value of } \Delta t$$

This can be read as the difference in the $f(t)$ direction divided by the difference in the t direction. When the step Δt is small enough, then this is indeed an approximation of the slope of the function at the point t_0 and therefore an approximation of $f'(t_0)$.

Task Create a script in R to compute the numerical derivative via the **2-point forward difference formula** for the data set used earlier and for which:

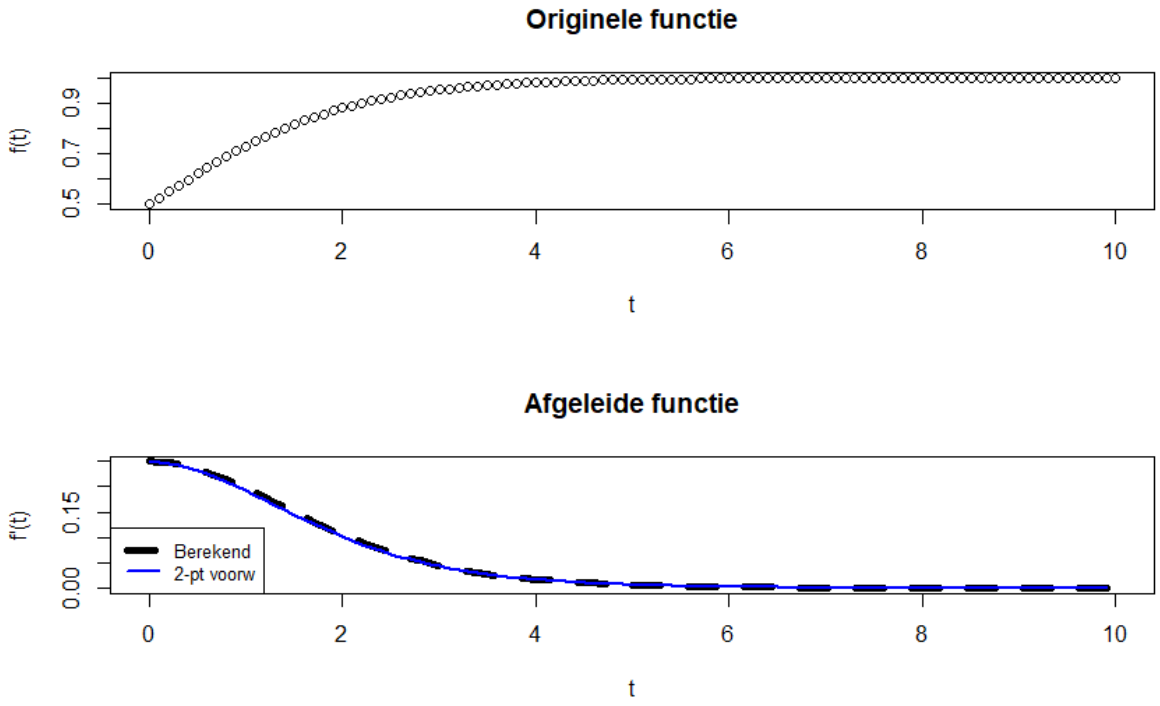
```
t <- seq(from=0, to=10, by=0.1)
```

```
ft <- function1(t)
```

Plot the result where the graph meets the following requirements:

- The original dataset is a scatter plot. This graph is on top and is titled 'Original function'.
- The result of the numerical derivative is below the graph of the original data set, and is plotted as a line without points and with thickness 3. The title of this graph 'Derivative'.
- The values of the derivative calculated by you are plotted in the same graph as the result of the numerical derivative through a dashed line of thickness 5.
- Add in the lower graph a legend to make the distinction between the curve of the derivative and the result calculated by the 2-point difference formula
- The x axis of the two graphs is called t.
- The y axis of the top graph is called $f(t)$, and of the bottom graph $f'(t)$

The resulting graphs are similar to:



Hint 1: Use a `for` loop.

Hint 2: Make sure in plotting that the sequences have the same length.

Hint 3: You can place multiple diagrams in a $m \times n$ matrix using the following command (calling *before* you plot the graphs): `par(mfrow=c(m,n))` .

Hint 4: The `lines` function enables you to add lines to an existing plot.

Hint 5: A legend can be added with the command `legend` .

The 3-point difference formula

After having determined the two-point forward derivative, you will determine the derivative with the 3-point difference formula. The 3-point forward difference formula is a combination of the two-point *forward* method and the 2-point *backward* method (we have not dealt with the backward method, but this is similar to the forward method).

In the 3-point difference method, the derivative of a function at the point t_0 can be approximated by:

$$f'(t_0) \approx \frac{f(t_0 + \quad t) - f(t_0 - \quad t)}{2 \quad t}, \quad \text{for small positive value of} \quad t$$

Task

Extend your script in R with the computation of the derivative via the 3-point difference method and plot the result in the *same* graph as the result of the two-point difference method and the curve of the derivative calculated by you. Is there a difference?

- Don't forget to adjust the legend.

Hint 1: Do the first and last time in the data set play a role in the computation of the derivative with the 3-point difference method?

Hint 2: How many points there are now superfluous in the original t data?

A noisy signal: 2-point and 3-point difference formulas

In previous assignments you have computed numerical derivatives via the two-point forward and the 3-point differences method for the function $f(t) = \frac{1}{1+e^{-t}}$. You have plotted the original function and you have plotted the results of various numerical derivatives together in one diagram beneath it. It is true that the given function $f(t)$ is a 'decent' function. A real dataset, however, often contains noise. In the tasks below you will investigate the effect of noise on the numerical derivatives.

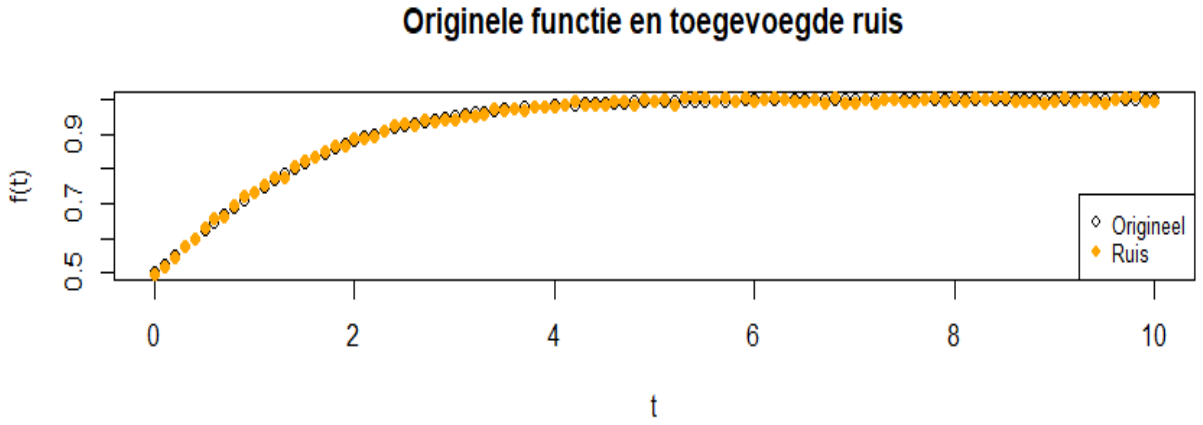
Task Extend your script in R with a function that adds noise to the function $f(t)$ and plot the result together with the original curve of the function $f(t)$ in one diagram.

Please ensure that:

- you plot the noisy signal in a different colour than the signal without noise.
- the original form of the equation is still visible after addition of the noise. So make the signal not too noisy;
- you adjust the legend of the diagram;

- plotting of the 2nd graph is temporarily suppressed.

The final diagram will look like the following:



Hint 1: Use the `jitter` command to introduce noise.

Hint 2: Use the `points` command after the `plot` command to add a scatter plot to the original graph .

Hint 3: You can play with the values within the `jitter` command. Make sure that the original form of the function is still recognizable.

Hint 4: You can avoid the plotting of the second graph by transforming that piece of code into 'comment lines' by placing '#' at the beginning of the sentences. This you can also do by selecting a whole piece of code and then enter "ctrl + shift + c".

Task Apply now the 2-point and 3-point difference methods to compute the numeric derivative of the signal with noise. Does one of the methods give a better result?

- Make R functions of the 2-point and 3-point methods. Then you can call these R functions for any t data and $f(t)$ data as input arguments.

Hint 1: Create *within* the function for computing a 2-point and 3-point based derivative an empty sequence for the storage of the results. Make sure you do this *outside* the `for` loop.

Hint 2: If the numerical derivatives are outside the y range of the graph: Reduce the noise on the signal, then the results can be compared with each other more easily.

Task Investigate the effect of changing the step size in t data and the increase/ decrease of the noise.