Probability Properties and Rules

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A. Information for lecturers

Unit description

- **Short description of the unit:** inquiry based introduction into probability properties and probability rules via subsequent tasks.
- **Expected prior knowledge of students:** random trial, probability space and classical probability definition.
- **The course and context in which it has been used in HE practice:** the unit was used during the lecture within Statistics I course. This course is in the third semester of bachelor degree of study programmes in the area of Business and Economics. Prerequisite for this course is one semester course of Mathematics.
- **Estimated duration:** Expected time for an activity during a lecture are 20-30 minutes depending on how much time one wants to give students for generating and using ideas, and on the length and deepness of classroom discussion one prefers or has time for.

Learning objectives

Subsequent tasks should

- develop conceptual understanding of: equally likely outcomes, certain event, impossible event, complement of event, mutually exclusive events, joint event and union of events. This understanding is based on an example of random experiment on rolling two differently coloured tasks.
- inspire to inquire about basic properties of probability and rules of probability in the context of previous task
- link the understanding of properties and rules of probability within an example of two dice with a general theorem.

IBME character

Students in small groups execute particular tasks and formulate findings and conclusions. The activity could be characterized as structured inquiry consisting of Exercise 1 and Exercise 2. Via fulfilling tasks given in an Exercise 1 (pictures of sample space are at work sheet) students are expected to develop answers on open questions in an Exercise 2. Further

they are supported to discuss together and defend their answers. Afterwards, the general theorem is performed by a teacher.

Mathematical content

The unit is meant as an activity which should introduce probability properties and probability rules.

Technological pedagogical content knowledge

This activity gives students first-hand experience of an exploration of probability properties with concrete example they can easily imagine and to „play“ with through pictures in a worksheet.

Learning path

At the beginning, the sample space of all possible outcomes is determined and consequently visualised. For some students, only the visual diagrams allows to proceed with further tasks. Then follows the playing with events which is straightforward and provides understanding to the nature of relationships among events within a concrete sample space (Exercise 1). This allows students to investigate general properties and relationships among events as is requested in Exercise 2. The consequent discussion expects students to back-up their answers. This brings them to generalise their findings from concrete example to general properties and rules.

Experiences

Though the unit was implemented at the end of the lecture students were engaged a lot. Also students who have already been distracted were again „back on the board“. Expressing events visually in diagrams (Exercise 1) was easy, thus also those who felt lost during the lecture, were engaged again. Exercise 2 was more challenging, however based on the graphical outcomes of the first exercise, students were able to proceed. Majority of them (cca75%) were making an effort to answer open questions in Exercise 2. The discussions in small groups to back-up their findings helped also those students who gave up before.

Student with special needs

We recommend to provide work sheets ahead of time in an electronic editable format, so students with print disabilities can work with them on the laptop/tablet or another computer-based device. It is better to give students a choice to work in groups or individually as some of them can prefer to discuss their observations and findings, but another are glad to work alone without frequent interruptions by others.

The unit, especially Exercise 1, is not directly accessible for visually impaired students for the sake of graphical diagrams. However, diagrams of sample space with dice symbols can be replaced by an editable table, where in the cells could be numbers representing outcomes of rolling the dice, such that 11|21|31|41|51|61.

B. Student learning activities

Tasks

Exercise 1. *(attached „dice_sheet_A4.pdf“ worksheet required) Experiment consists of a roll of two dice (the first is marked in red, the second is marked in black). Determine the sample space and calculate the probability of events A to I based on the definition of the classical probability ($P(A) = m(A)/m(\Omega)$):*

a.) event A: the sum of the dots on both dice will be 10.

b.) event B: the sum of the dots on both dice won't be 10.
Additionally, formulate the probability of event B using event A.

c.) event C: outcome of rolling dice is the same number on both dice.

d.) event D: outcome of rolling dice is „5“ on both dice.

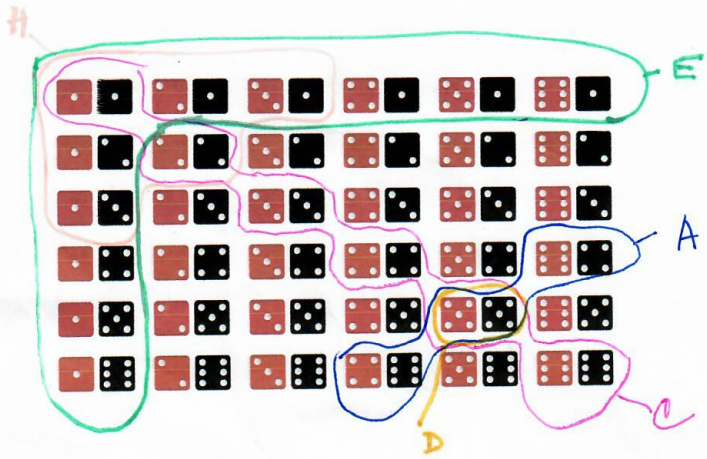
e.) event E: outcome of rolling dice is „1“ at least on one dice.

f.) event F: the sum of the dots on both dice will be 10 or at least on one dice will be „1“.
Additionally, formulate the probability of event F using the event A and E.

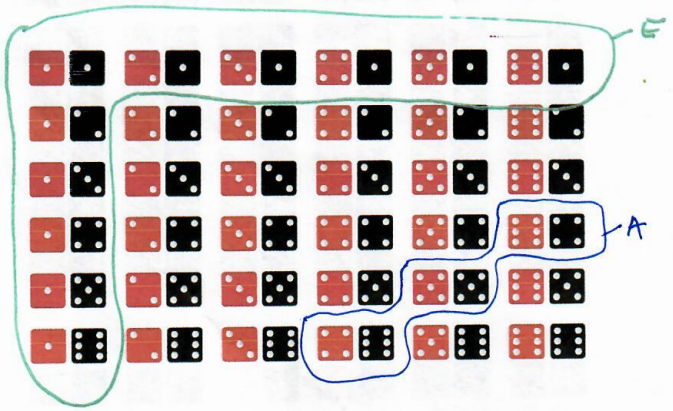
g.) event G: the sum of the dots on both dice will be 10 or the outcome of rolling dice will be the same number on both dice.
Additionally, formulate the probability of event G using event A and C.

h.) event H: the sum of the dots on both dice will be maximally 4.

i.) event I: the sum of the dots on both dice will be maximally 4 or the outcome of rolling dice will be the same number on both dice or there will be „1“ at least on one dice.
Additionally, formulate the probability of event I using events H, C a E.

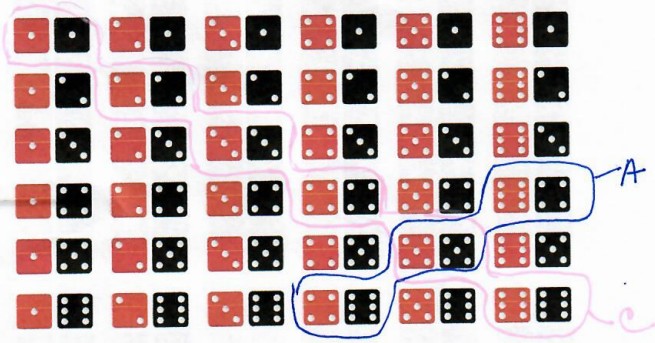


A: součet je deset
C: na obou kostkách stejně.
E: na asp. jedné kostce jedenáct
H: součet je nejvýše 4
D: dvě „pětky"



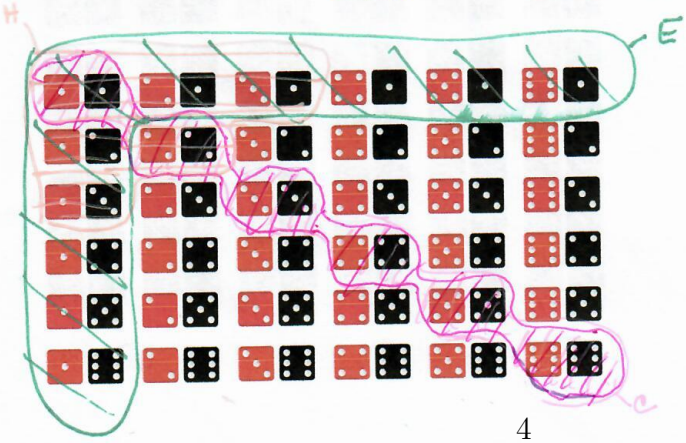
$A \cup E$

$m(A) = 3$
$m(E) = 11$
$m(A \cap E) = 0$



$A \cup C$

$m(A) = 3$
$m(C) = 6$
$m(A \cap C) = 1$



$H \cup C \cup E$

$m(H) = 6$
$m(C) = 6$
$m(E) = 11$
$m(H \cap C) = 2$
$m(H \cap E) = 5$
$m(C \cap E) = 1$
$m(H \cap C \cap E) = 1$

Exercise 2. *Based on the example 1 (attached „dice_solution.pdf“ and „Exercise_2_probability_rules_sheet.pdf“ required), try to replace the question mark symbol in the following relationships.*

i. $P(A) + P(A') =$ *?*

ii. *If* $A_1 \subseteq A_2$, *then* $P(A_1)$ *?* $P(A_2)$
Take inspiration, for example, from events D and A.

iii. *If* $A_1 \bigcap A_2 = \emptyset$, *then* $P(A_1 \bigcup A_2) =$ *?*
Take inspiration, for example, from events A and E.

iv. *If* $A_1 \bigcap A_2 \neq \emptyset$, *then* $P(A_1 \bigcup A_2) =$ *?*
Take inspiration, for example, from events A and C.

v. $P(A_1 \bigcup A_2 \bigcup A_2) = P(A_1) + P(A_2) + P(A_3)$ - *?* - *?* - *?* + *?*
Take inspiration, for example, from events H, C and E.

Worksheets and files

- dice_sheet_A4.pdf
- dice_solution.pdf
- Exercise_2_probability_rules_sheet.pdf

Time needed

- 2 minutes: deriving sample space
- 5 minutes: graphically expressing events from Exercise 1
- 8 minutes: Exercise 2
- 5 minutes: discussion about arguments for answers in Exercise 2