

Introduction to Derivatives

Zuzana Pátíková; Brno University of Technology

A. Information for lecturers

Unit description

Description: The main aim of the lesson is to introduce the concept of the derivative of a real function of one real variable at a point with an emphasis on understanding its geometrical interpretation. The warm-up activities contain recalling the notions of slope of a line and a tangent line (Task 1 and 2). Then definition of the derivative is presented in terms of a limit and interpreted geometrically as a slope of a tangent line. In Task 3 students compare and combine characteristics of both approaches which helps them to strengthen their perception of relationship between these two approaches. In Task 4 students discover the relationship between a sign of the derivative and monotonicity properties of the function. Working with worksheets where the Tasks are defined can be individual/in pairs/in small groups.

Student and discipline level: 1st year bachelor students of technical/engineering study programs

Prior knowledge: Expected student and lecturer knowledge and skills are

- notion of a function, value of a function at a point and graph of a function
- slope of a line
- linear function and a graph of linear function
- tangent line
- calculation of limits

Estimated duration: 100 min

Facilities: worksheets

Learning objectives

At completion of the unit, students will be able to

- talk about and work with the concept of the derivative of a function at a point
- understand why one would be interested in the derivative of a function at a point and that it is an effective tool for finding the slope of the tangent line
- understand how the sign and value of the derivative is related to monotonicity properties of the function
- understand definition of the derivative in terms of limits
- compute derivatives of simple functions numerically using the limit process
- decide critically about what comes as an outcome of differentiation
- explore and formulate ideas about slope of a line and derivative of a function at a point in collaboration with peers, verify and present findings to others

IBME character

The teaching unit contains elements of guided and structured inquiry. Activities follow each other in order to build up the concept of the derivative of a function at a point and main ideas behind this

notion properly. All four tasks are based on work with worksheets containing graphs of linear and/or various other functions.

Mathematical content

- slope of a line
- tangent line
- derivative of a function at a point
- monotonicity
- visualization and geometrical interpretation of a derivative

Learning path

1. Teacher presents on the board in discussion with students: Review of analytical expression of a linear function (=linear equation) and interpretation of the slope coefficient in the graph of the line.
2. Task 1: Match pictures of graphs of lines with the values of their slopes (10 pairs).
3. Plenary discussion of the results.
4. Task 2: Find the points with a prescribed value of the slope of tangent line in the graph of a given function.
5. Teacher presents on the board: Definition of the derivative of a function at a point.
6. Illustration of the introduced notion of the derivative through an interactive visualisation in Simreal+ (available online https://grimstad.uia.no/perhh/phh/MatRIC/SimReal/no/SimRealP/AA_sim/Mathematics/DiffInt/Diff/Scalar/v/P/Diff.swf).
7. Teacher presents on the board: Illustration of the definition for the function $f(x) = 2x$.
8. Task 3: Given the function ($f(x) = x^2$) and its graph and five points, estimate the slope of the tangent line at given points and then calculate the derivative using the definition.
9. Plenary discussion of results and review of calculations on the board.
10. Task 4: Given a graph of a function, estimate/determine where the derivative is zero, positive or negative and guess/find relation between the sign of the derivative and the behavior of the function in terms of monotonicity properties.

Experiences

The piloting lesson has been realized at Faculty of Technology, Tomas Bata University in Zlín within two small study groups of 11 and 9 students in autumn semester 2019. The small number of students brings an advantage in several aspects:

- teacher has a good notion of math skills of individual students,
- work in small groups (2-3 students) is easily arranged,
- teacher can walk around the class and support individuals/small groups,
- work with technology can be easily arranged and managed.

Teaching format of the subject (Mathematics 1) is a seminar, not a lecture and exercises/tutorials. The teaching unit has been used as a combination of presentation of theory with practical tasks. This fact has influenced structure of the teaching unit. During the warm up activity in Task 1, which was supposed to be only quick revision, the class broke up into two groups. In the first one the students succeeded in completing the task individually, whereas the students in the second group struggled with the basic notions of a function, value of a function, etc., and had troubles to apply the presented review of the theory about the slope of a line to the task. To overcome this situation, the students were mixed and coupled afterwards, so that those who finished earlier helped the ones who

needed help. The couples have been kept for the rest of the class with the possibility to communicate with anybody else. Task 2 revealed that some students did not know how to construct (sketch) a tangent line. In Tasks 3 and 4 students worked quite well, however at different paces. Those who were working faster were asked to do the calculations on the board for the sake of checking (Task 3) or to join other groups to supervise/support them. Overall, the issue of different levels of the students' prior knowledge and skills influenced the pace of the lesson. During the individual or group work, the teacher walked around the class and managed the students in order to use their capacity in explaining and sharing knowledge, not to let them get bored, so that the students were able to provide/receive help when needed, to overcome difficulties etc.

We think that the structure of the teaching unit has been well designed in order to achieve the goal of the lesson: to link the perception of the notion of the derivative to a geometrical interpretation.

Student with identified needs

The worksheets are prepared in MS Word (.doc) format, so the size and colour of the text can be adjusted. The graphs in a picture format (.jpg) can only be enlarged but not thickened.

Assessment

Suggestions: an optional assessment could consist of a short reviewing task in the beginning of the next lesson so that students recall and recap the concept of the derivative at a point and strengthen their knowledge.

Relevance of/to the real world

Derivative of a function is needed in science wherever rate of change of a quantity appears (e.g. in physics, chemistry, biology, engineering, medicine). Further, derivative of a function is an important concept in optimization.