

Teaching unit Multivariate calculus

Course:

Mathematics 2 (8 ECTS credits, 1st year graduate students , 3 groups of 20-25 students), week 4

Aim:

To foster mathematical and geometric intuition for understanding basic concepts of multivariate calculus (contour plot, partial derivatives, stationary points, extreme points, etc.)

Activity 1: The revision of the basic terms (contour plot, partial derivatives)

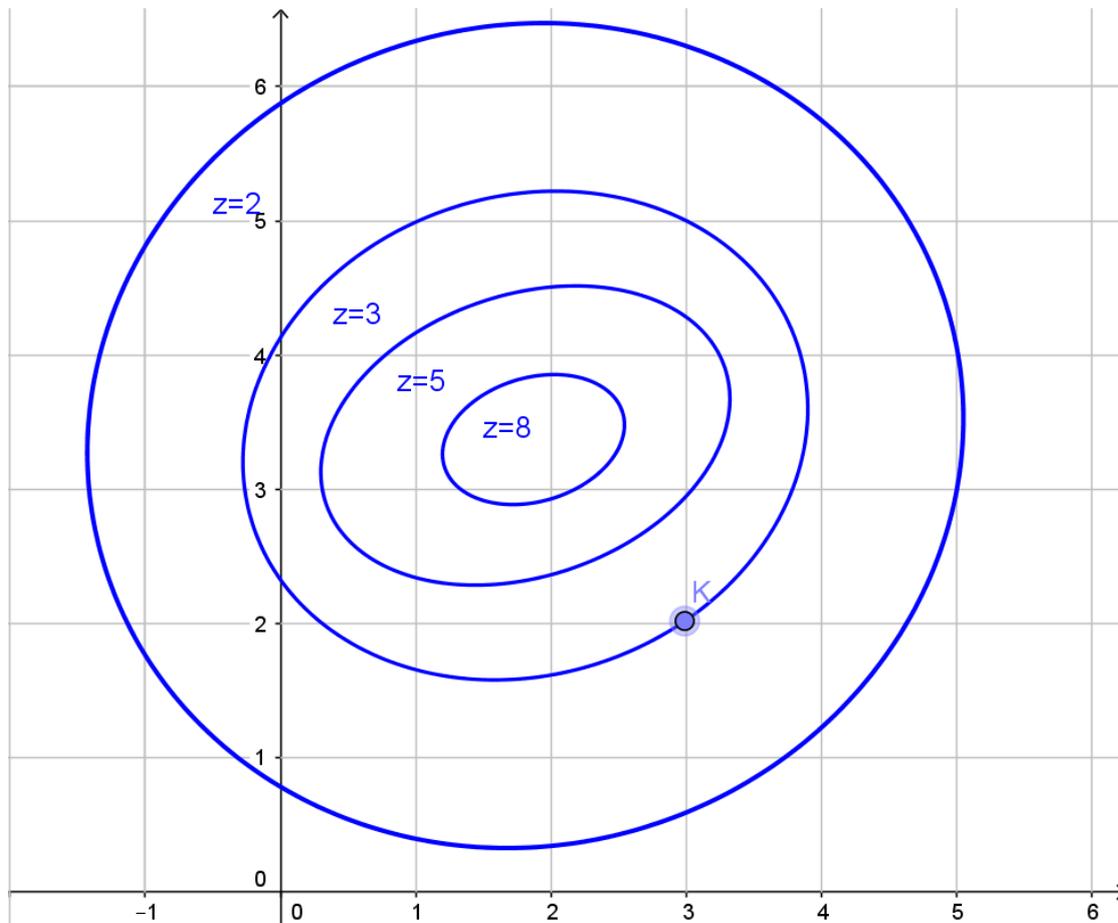
Answer the questions 1, 2 of the quiz on Classroom voting system Socrative (room BE483461)

Problem 1:

The figure depicts the contours of the function $z=f(x,y)$. Find the solution of the equation $f(x,5)=3$.

Problem 2:

Suppose that the function $z=f(x,y)$ has both partial derivatives in the point $K=[3,2]$. Try to determine their signs.

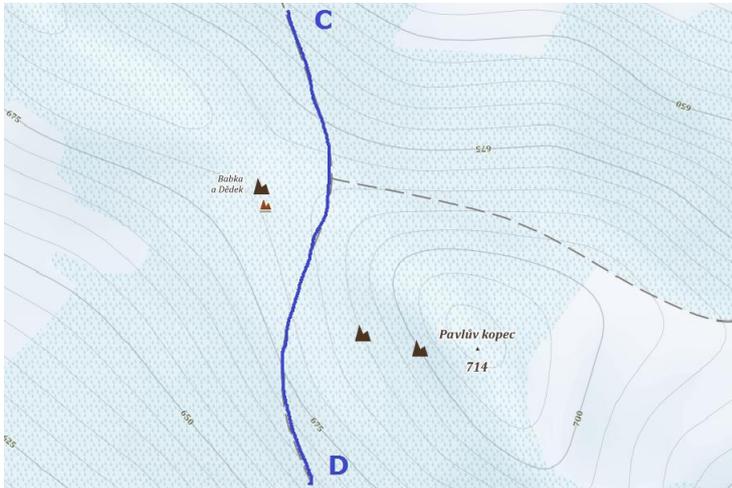


Activity 2: Multivariate optimization

Solve the problems 1-3 in groups of three or four (1 handout for each group). Completing the tasks is followed by the discussion.

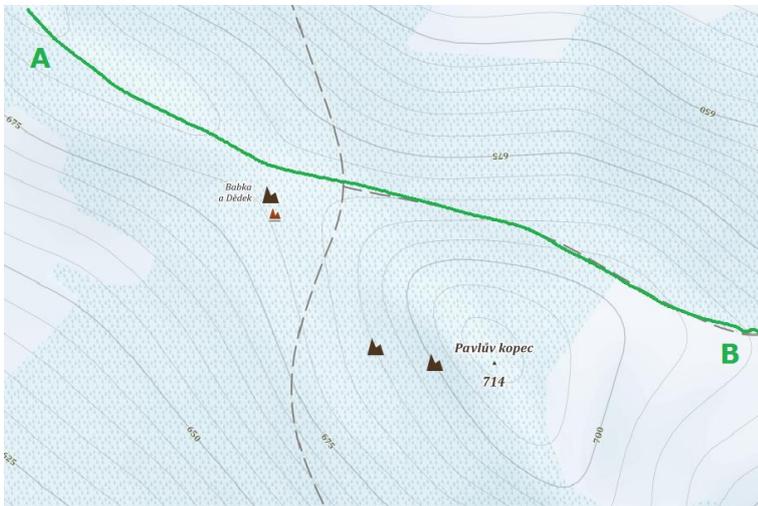
Problem 1:

Find the lowest and the highest point on the path from C to D



Problem 2:

Find the lowest and the highest point on the path from A to B



Supplementary questions:

- can you see any extrema points on the map?
- describe the relation between the density of contours and „the steepness of the surface“.
- why is the highest place on the path CD and the lowest place on AB represented by the same point?
- can you identify a saddle point on the map?

Problem 3:

Find the lowest and the highest point in the selected region.



Supplementary questions:

- does the maximum lie inside the region or on the boundary?
- does the minimum lie inside the region or on the boundary?
- are both partial derivatives equal to zero in the extrema points?
- try to characterize the possible locations of extrema points .