Open Autograph. On the dropdown menu open a "New Complex Number Page".

Open each task on a New Complex Number Page as you work through them.

Do not save your work. Re-load a task on a new page if necessary.

When asked what you notice, or to explore a task, you may want to consider situations when z is real or imaginary, the modulus or argument of z, symmetries, etc.

<u>Task 1:</u> Open the Autograph file *Task 1*

There are three complex numbers labelled z_1, z_2 and z. z_1 is to be kept fixed while z_2 and z can be moved. Select z_2 and move it until z reaches the position $6+5j$.

(a) What complex number is z_2?
Right click and "Unhide All" to check your answer. The correct answer appears in green.

(b) What is the mathematical relationship between z_1, z_2 and z (how are they connected)?

(c) Now calculate by hand: With $z_1 = -3 + j$ and $z = 6 + 5j$, find z_2 such that $z_1 + z_2 = z$.

(d) Re-load *Task 1*. Move z_2 around the screen and notice how z changes. Describe the position of z in relation to z_1 and z_2.

(e) Explore this relationship. Move z_1 and z_2 to different locations but make sure that z still ends up being $6 + 5j$. Does what you thought in (d) still hold?

<u>Task 2:</u> Open the Autograph file *Task 2*

There are three complex numbers labelled z_1, z_2 and z. z_1 is to be kept fixed while z_2 and z can be moved. Select z_2 and move it until z reaches the position $3+j$.

(a) What complex number is z_2?
Right click and "Unhide All" to check your answer. The correct answer appears in green.

(b) What is the mathematical relationship between z_1, z_2 and z (how are they connected)?

(c) Now calculate by hand: With $z_1 = -1 - 3j$ and $z = 3 + j$, find z_2 such that $z_2 - z_1 = z$.

(d) Re-load *Task 2*. Move z_2 around the screen and notice how z changes. Describe the position of z in relation to z_1 and z_2.

(e) Explore this relationship. Move z_1 and z_2 to different locations but make sure that z still ends up being $3 + j$. Does what you thought in (d) still hold?

<u>Task 3:</u> Open the Autograph file *Task 3*

There are two complex numbers on the screen: $z_1 = -3 - j$ and $z_2 = 1 - j$.

(a) Calculate z_1 multiplied by z_2 (by hand).
Right click and "Unhide All" to check your answer. The correct answer appears in green.

(b) Calculate (by hand) a new value of $z_1 z_2$ by keeping z_1 and changing $z_2 = -1 - j$.

(c) Select z_2 and move it to the new position $-1 - j$. Read off the result for $z = z_1 z_2$.
Were you correct in (b)?

(d) Now calculate (by hand) a new value of $z_1 z_2$ for $z_1 = z_2 = -1 - j$.

(e) Select z_1 and move it to the new position $-1 - j$. Read off the new result for the number z. Were you correct with your calculation in (d)?

(f) Explore this task by choosing your own values for z_1 and z_2. Multiply them by hand and check your answer using the Autograph file.

<u>Task 4:</u> Open the Autograph file *Task 4*

There are three complex numbers labelled z_1, z_2 and z. The complex number $z_1 = -2 + j$.

(a) What is the complex conjugate of z_1 (usually denoted by z_1^*)?

(b) Select z_2 and move it to the position of the complex conjugate of z_1. Notice what is happening to z. What is the mathematical relationship between z_1, z_1^* and z (how are they connected)?

(c) Verify this by hand (a calculation).

Task 5: Open the Autograph file *Task 5*

There are two complex numbers labelled z_1 and z with $z = z_1^2$.

(a) Select z_1 and move it to the new position $3 + j$. Notice how z changes.

(b) Calculate (by hand) a new value for $z = z_1^2$ when $z_1 = 3 + 2j$.

(c) Select z_1 and move it to the position $3 + 2j$ to check your answer. Were you correct?

(d) Now move z_1 to the position $1 + j$. Interpret the result.

(e) Select z_1 and move it until $z = z_1^2$ is real. Find different z_1 so that z_1^2 is real. What property must z_1 have so that z_1^2 is real?

(f) Select z_1 and move it until $z = z_1^2$ is purely imaginary and negative. What property must z_1 have so that z is purely imaginary and negative?
When does $z = z_1^2$ become purely imaginary and positive?

Task 6: Open the Autograph file *Task 6*

There are two complex numbers labelled z_1 and z_2.

(a) Select z_1 and move it to different positions. There is a (mathematical) relationship between z_1 and z_2 but it is quite hard to see - so first move z_1 so that z_1 is real. What do you notice about z_2?

Try different places for z_1 keeping it always a real number. When does z_2 have a larger modulus than z_1? When does it have a smaller modulus? When do they both have the same modulus? Remember to also try negative value for z_1.

(b) Try to find a relationship between the modulus of z_1 and the modulus of z_2.

(c) Click on the polar co-ordinate icon on the toolbar. Now allow z_1 to take any value, not only just real. Move z_1 and focus on the angle that it makes with the positive real axis. Also focus on the angle that z_2 makes with the positive real axis. Try to find a relationship between the angles as you move z_1 around.

(d) What do you think is the (mathematical) relationship between z_1 and z_2?