\documentclass[12pt]{book}

\renewcommand{\familydefault}{\sfdefault}

\usepackage{graphicx,tabularx,fullpage,float,extsizes,tikz,pgfplots,amsmath,booktabs,cancel,lscape,wrapfig,sfmath,colortbl,sidenotes}

\usepackage[bottom=1in,left=1in,right=1in,marginparwidth=1in,marginparsep=-0.5in]{geometry}

\usetikzlibrary{arrows,calc}

\pgfplotsset{compat=newest}

\renewcommand\arraystretch{1.3}

\pgfplotsset{major grid style={blue!50!white}}

\pgfplotsset{minor grid style={blue!25!white}}

\begin{document}

\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\vspace{3mm}

\center

Open Autograph. On the dropdown menu open a "New Complex Number Page".\\

\vspace{0.5cm}

Open each task on a New Complex Number Page as you work through them.\\

\vspace{0.5cm}

Do not save your work. Re-load a task on a new page if necessary. \\

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When asked what you notice, or to explore a task, you may want to consider situations when $z$ is real or imaginary, the modulus or argument of $z$, symmetries, etc.\\

% Addition of CNs

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\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\noindent{\underline {\bf {Task 1:}}} \; Open the Autograph file \emph{Task 1}

\begin{enumerate}

\item[]

There are three complex numbers labelled $z\_1, z\_2$ and $z$. \hspace{0.05cm} $z\_1$ is to be kept fixed while $z\_2$ and $z$ can be moved.

Select $z\_2$ and move it until $z$ reaches the position $6+5j$.

\item [(a)] What complex number is $z\_2$? \\ Right click and ``Unhide All'' to check your answer. The correct answer appears in green.

\item [(b)] What is the mathematical relationship between $z\_1, z\_2$ and $z$ (how are they connected)?

\item [(c)] Now calculate by hand: With $z\_1=-3+j$ and $z=6+5j$, find $z\_2$ such that $z\_1 + z\_2 = z$.

\item [(d)] Re-load \emph{Task 1}. Move $z\_2$ around the screen and notice how $z$ changes. Describe the position of $z$ in relation to $z\_1$ and $z\_2$.

\item [(e)] Explore this relationship. Move $z\_1$ and $z\_2$ to different locations but make sure that $z$ still ends up being $6+5j$. Does what you thought in (d) still hold?

\end{enumerate}

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% Subtraction of CNs

\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\noindent {\underline {\bf {{Task 2:}}} \; Open the Autograph file \emph{Task 2 }

\begin{enumerate}

\item[]

There are three complex numbers labelled $z\_1, z\_2$ and $z$. \hspace{0.05cm} $z\_1$ is to be kept fixed while $z\_2$ and $z$ can be moved.

Select $z\_2$ and move it until $z$ reaches the position $3+j$.

\item [(a)] What complex number is $z\_2$? \\

Right click and ``Unhide All'' to check your answer. The correct answer appears in green.

\item [(b)] What is the mathematical relationship between $z\_1, z\_2$ and $z$ (how are they connected)?

\item [(c)] Now calculate by hand: With $z\_1=-1-3j$ and $z=3+j$, find $z\_2$ such that $z\_2 - z\_1 = z$.

\item [(d)] Re-load \emph{Task 2}. Move $z\_2$ around the screen and notice how $z$ changes. Describe the position of $z$ in relation to $z\_1$ and $z\_2$.

\item [(e)] Explore this relationship. Move $z\_1$ and $z\_2$ to different locations but make sure that $z$ still ends up being $3+j$. Does what you thought in (d) still hold?

\end{enumerate}

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% Multiplication of CNs

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\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\noindent {\underline {\bf {{Task 3:}}} \; Open the Autograph file \emph{Task 3}

\begin{enumerate}

\item[]

There are two complex numbers on the screen: $z\_1 = -3-j$ and $z\_2 = 1-j$.

\item [(a)] Calculate $z\_1$ multiplied by $z\_2$ (by hand).\\

Right click and ``Unhide All'' to check your answer. The correct answer appears in green.

\item [(b)] Calculate (by hand) a new value of $z\_1 z\_2$ by keeping $z\_1$ and changing $z\_2 = -1-j$.

\item [(c)] Select $z\_2$ and move it to the new position $-1-j$. Read off the result for $z = z\_1 z\_2$. \\ Were you correct in (b)?

\item [(d)] Now calculate (by hand) a new value of $z\_1 z\_2$ for $z\_1 = z\_2 = -1-j$.

\item [(e)] Select $z\_1$ and move it to the new position $-1-j$. Read off the new result for the number $z$. Were you correct with your calculation in (d)?

\item[(f)] Explore this task by choosing your own values for $z\_1$ and $z\_2$. Multiply them by hand and check your answer using the Autograph file.

\end{enumerate}

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% Complex conjugate

\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\noindent {\underline {\bf {{Task 4:}}} \; Open the Autograph file \emph{Task 4}

\begin{enumerate}

\item[]

There are three complex numbers labelled $z\_1, z\_2$ and $z$. The complex number $z\_1 = -2+j$.

\item [(a)] What is the complex conjugate of $z\_1$ (usually denoted by $z\_1^\*$)?

\item [(b)] Select $z\_2$ and move it to the position of the complex conjugate of $z\_1$. Notice what is happening to $z$. What is the mathematical relationship between $z\_1, z\_1^\*$ and $z$ (how are they connected)?

\item [(c)] Verify this by hand (a calculation).

\end{enumerate}

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% Squaring of a CN

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\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\noindent {\underline {\bf {{Task 5:}}} \; Open the Autograph file \emph{Task 5}

\begin{enumerate}

\item[]

There are two complex numbers labelled $z\_1$ and $z$ with $z=z\_1^2$.

\item [(a)] Select $z\_1$ and move it to the new position $3+j$. Notice how $z$ changes.

\item [(b)] Calculate (by hand) a new value for $z=z\_1^2$ when $z\_1 = 3+2j$.

\item [(c)] Select $z\_1$ and move it to the position $3+2j$ to check your answer. Were you correct?

\item [(d)] Now move $z\_1$ to the position $1+j$. Interpret the result.

\item [(e)] Select $z\_1$ and move it until $z=z\_1^2$ is real. Find different $z\_1$ so that $z\_1^2$ is real. What property must $z\_1$ have so that $z\_1^2$ is real?

\item [(f)] Select $z\_1$ and move it until $z=z\_1^2$ is purely imaginary and negative. What property must $z\_1$ have so that $z$ is purely imaginary and negative? \newline

When does $z=z\_1^2$ become purely imaginary and positive?

\end{enumerate}

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% Cubing of a CN

\noindent {\bf Module Title: \hfill Tutorial: Complex Numbers}

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\noindent {\underline {\bf {{Task 6:}}} \; Open the Autograph file \emph{Task 6}

\begin{enumerate}

\item[] There are two complex numbers labelled $z\_1$ and $z\_2$.

\item [(a)] Select $z\_1$ and move it to different positions. There is a (mathematical) relationship between $z\_1$ and $z\_2$ but it is quite hard to see - so first move $z\_1$ so that $z\_1$ is real. What do you notice about $z\_2$?

Try different places for $z\_1$ keeping it always a real number. When does $z\_2$ have a larger modulus than $z\_1$? When does it have a smaller modulus? When do they both have the same modulus? Remember to also try negative value for $z\_1$.

\item [(b)] Try to find a relationship between the modulus of $z\_1$ and the modulus of $z\_2$.

\item [(c)] Click on the polar co-ordinate icon on the toolbar. Now allow $z\_1$ to take any value, not only just real. Move $z\_1$ and focus on the angle that it makes with the positive real axis. Also focus on the angle that $z\_2$ makes with the positive real axis. Try to find a relationship between the angles as you move $z\_1$ around.

\item [(d)] What do you think is the (mathematical) relationship between $z\_1$ and $z\_2$?

\end{enumerate}

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