Considerations and inferences. Structure and types of theorems

Yuriy Mazhuga; Boris Grinchenko Kyiv University

A. Information for lecturers

Unit description

Description: What is more important to teach - knowledge or understanding? All my teaching practice and experience show that it is necessary to teach understanding. One of the psychological factors which is significantly impedes the mastering of theoretical material from mathematics, is a large number of determinations, formulations, theorems, properties, formulas. The students coming for high schools are already familiar with many rules, theorems of algebra and geometry, and they capable to reproduce and prove them. However, they have great difficulty when dealing with the solution of new unknown problems (tasks). Based on previous knowledge, in framework of module "Elements of Mathematical Logic" students learn to use basic concepts, determine type of statements, apply logical operations to propositions, construct deductive and inductive considerations, formulate and prove of theorems. Below is the concrete task that students are invited to consider in the practical class.

The following picture is given:
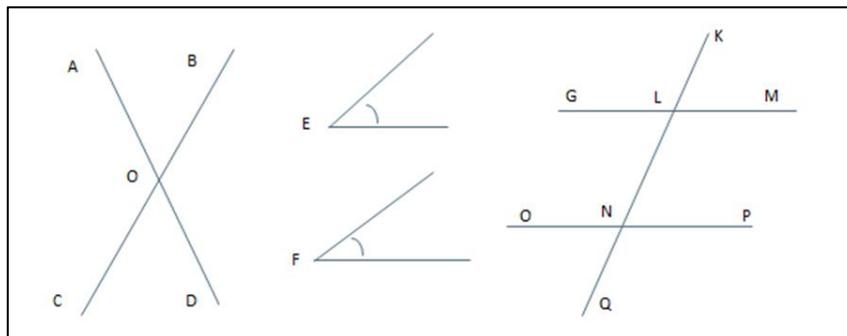
What propositions (theorems) can be formulated using the presented images?
What is the structure of these statements?
Which of them will be true, and which be false?
How can these propositions be written as formulas?

Students are invited to discuss possible approaches to solving the task with partners in small groups (3-4 persons). Based on the analysis of the statements made, using their previous knowledge, the students themselves come up with answers to the questions, formulate their questions to the topic, suggest appropriate regulatory cards with each stage. The answers and results which students offer are discussed in classroom. The activity leads to the research. Students partly independently, partly with the teacher's advice, express assumptions, formulate hypotheses, make generalizations and conclusions, carry out proofs, learn further to apply the new knowledge for concrete examples during a practice session.

Student and discipline level: This unit is concerned with elements of Mathematical Logic, that would fit in all bachelor's programs, but it was designed for use with first year bachelor students of pedagogical institute.

Prior knowledge: Expected student and lecturer knowledge and skills are
- high school mathematics background;
- acquaintance with the basic concepts and operations in mathematical logic, type of statements, logical operations to statements.

Estimated duration: Expected time for activity during a practical class: approximately 40 minutes depending how much time one wants to give students for generating and using ideas, and on the length and deepness of classroom discussion one prefers or has time for. The implementation and application the new knowledge for concrete examples is 40 minutes during a practice session.

Learning objectives

At completion of the unit, students will be able to
- use basic concepts and operations in mathematical logic;
- construct clear, logical arguments;
- carry out operations on the statements;
- construct theorems, to determine their structure and types;
- prove theorems in direct and indirect ways;
- work independently and explore ideas in collaboration with partners;
- think more critically about and reflect on mathematical methods and techniques;
- share results to others.

IBME character

The class is new to inquiry. The interest in the study of mathematics is mostly average. The level of mathematical preparation of all students is formed on the basis of the math program of high school. However, practice shows that the real level is very different. The experience in managing the trajectory of one's own learning is practically absent.

The practical class can be characterized as structured inquiry. Students provide a mathematical explanation (proof) of the characteristic features of the theorems, their structure and types. The lecturer, as appropriate, asks questions to students to make such an explanation, encouraging, however, students to do this on their own. The lecturer coordinates the process of learning with the help of regulatory cards.

Mathematical content

The main mathematical content is
- mathematical concepts and mathematical sentences;
- statements and logical operation;
- structure and types of theorems;
- truth or falsity of statements.

Technological pedagogical content knowledge

The level technological support that promotes the formation of an efficient knowledge base for students involves: studying theoretical material and working on mathematical problems. Many

students have difficulties in learning mathematics. For them mathematics is a large number of determinations, formulations, theorems, properties, formulas.

In order to overcome these difficulties may be used the following techniques:
- simultaneous use of all constituents carrying the same content information: words, pictures, symbols, models, etc .;
- "dense packing" and visualization of knowledge, the record of learned material in the tables, diagrams.

Such work contributes to the activation of students' thinking, since it implies: the selection of the main and secondary, finding a common and different, establishing links and relationships between concepts. At the same time students better memorize the theoretical material and faster replicate it if necessary. When selecting training exercises, the level of their complexity, the availability of the appropriate number of typical tasks is taken into account.

It should also be noted that it is necessary to raise the basic level of mathematical competences of students, to find the optimal balance of theoretical and demonstration-visual material with the use of additional ICT tools.

Learning path

Students are invited to ask a question or make an observation about the task (picture above), the lecturer provides tips if appropriate. The activity during the class, in which students think about the characteristic features of the theorems, is an interactive discussion. By asking the questions, the students are expected to be interested and encouraged in the exploration of posed mathematical problem. Mostly, they do this not individually but with other students in groups. The goal is that students experience that by talking about mathematics with other, their own thinking becomes deeper and more meaningful.

The 5E model inquiry (Engage, Explore, Explain, Elaborate, Evaluate) is selected as the basis for developing a scenario for practical class. The following phases are presented in the table below:

Assignment	Activity	E-phasis
1	The teacher invites students to identify the characteristic features of the theorems, their structure and types. The geometrical images are used for more effective student engagement.	Engage
2	Based on the analysis of the statements made, using their previous knowledge, the students themselves (possibly following the teacher's instructions) come up with answers to the questions: - which statements are defined as theorems; - what are the characteristic features and the structure of the theorems; - what are the types of theorems; - how to determine the truth or falsity of such statements. The teacher coordinates the process of studying students with the help of regulatory cards.	Explore
3	Students provide a rigorous mathematical explanation (proof) of the characteristic features of the theorems, their structure and types. The teacher, as appropriate, asks questions to students to make such an explanation, encouraging, however, students to do this on their own.	Explain
4	Signs of the theorems, structure and types of theorems. Law of contraposition. Deductive reasoning.	Elaborate

	Writing statements using mathematical formulas. Students under the guidance of the teacher are actively working to consolidate their newly acquired new knowledge.	
5	Is the task complete? Do we have a clear idea of the signs of the theorems, the structure and types of theorems? Can we formulate relevant statements, write down using mathematical symbols, make generalization and proof? Students get new knowledge in the process of inquiry based activity. Formation of mathematical and general competences.	Evaluate

During the class such digital resources are used: Mentimeter, Padlet, PowerPoint, Kahoot.

Experiences

It was my first experience of using inquiry within HE classroom practice. But the interactive discussion in the class went sufficiently well, most students actively joined the solution of the questions and problems. All students stated that the work using the inquiry method enabled them to feel confident in their own strength while studying new material, encouraged them to work independently, and helped to update their knowledge. As a result, mathematical material became more accessible for memorization. The motivation for studying mathematics has changed.

At the end of each session, students were interviewed: "Was the activity useful? Why? Was it induced to research? How?". All answers were positive:
"Activity has made possible to feel self-confidence while studying new material".
 "There was an interest in obtaining the result of the study".
"There was an interest in experimenting with the given values to find out how their change will affect the result".
"A number of questions were raised to the subject of research."
"The activity was interesting, the task are non-standard".

It should also be noted that the implementation of inquiry based learning is much more effective in the case of classes with small groups of students (up to 10-15).

Student with special needs

I have not had such an experience yet.

Assessment

Suggestions: let students
- construct the other examples of similar reasoning and inferences;
- consider the simplest schemes of deductive considerations.

Relevance of/to the real word

Mathematical logic improves the conceptual understanding of mathematics and other sciences, forms the general competence of students.

B. Student learning activities

The lecture

The teaching and learning unit is based on interactive discussion of students and lecturer caused by the following task:

The following picture is given:
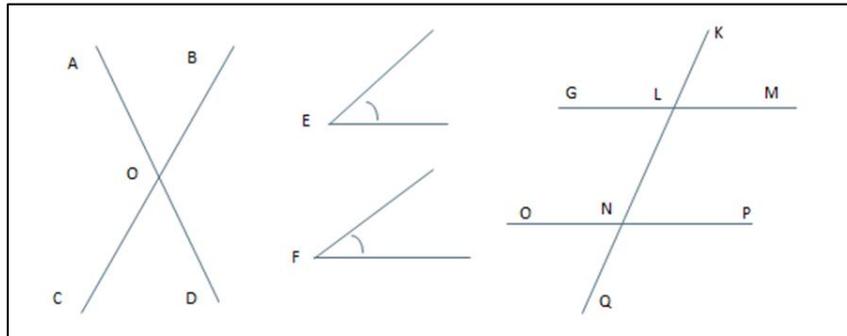
What propositions (theorems) can be formulated using the presented images?
What is the structure of these statements?
Which of them will be true, and which be false?
How can these propositions be written as formulas?

The unit consists of 5 assignments. The expected participation of students in each activity has already been indicated in the description of the learning path in the section *Information for lecturers.*

Assignment 1: Engage

Learning objective: Learning to ask questions correctly, formulate mathematical statements and expressions, determining their truth or falsity.

Student activity: Asking and answering questions, offering mathematical statements and propositions, determining their truth or falsity. For example:

"The angle AOB is equal angle COD". This proposition is true.
"If the angles are vertical, then they are equal". This proposition is true.
"If the angles are not vertical, then they are not equal." This proposition is false.
"The angle E is equal angle F". This proposition is true.
"The angle KLM is equal angle LNP". This proposition is true.
"If the angels are equal, then they are vertical." This proposition is false.
and so on.

Tool use: PowerPoint, Smart Board, regulatory cards, pen and paper, chalk and blackboard.

Assignment 2: Explore

Learning objective: Figuring out which statements are determined as theorems, their structure and type.

Student activity: Using previous statements and propositions students themselves, possibly following the teacher's instructions, choose (as an option) the following theorems and write them:

1) The theorem is given: *"If the angles are vertical (A), then they are equal (B)"*. It's true.

$$A => B$$

2) The inverse theorem: *"If the angels are equal, then they are vertical."* It's false.

$$B \Rightarrow A$$

3) The opposite theorem: *"If the angels are not vertical, then they are not equal."* It's false.

$$\overline{A} \Rightarrow \overline{B}$$

4) The theorem is inverted to the opposite: *"If the angles are not equal, then they are not vertical."* It's true.

$$\overline{B} \Rightarrow \overline{A}$$

Tool use: PowerPoint, Smart Board, regulatory cards, pen and paper, chalk and blackboard.

Assignment 3: Explain

Learning objective: Improving understanding of the concept of theorems and their properties.

Student activity: Providing explanation of characteristic features and proof of theorems, analysis of their structure and types. Making a conjecture (generalization) that theorems 1 and 4 (2 and 3) are equivalent.

Tool use: PowerPoint, Smart Board, regulatory cards, pen and paper, chalk and blackboard.

Assignment 4: Elaborate

Learning objective: Deepening previous experience and generating new knowledge.

Student activity: Establishing the equivalence of theorems $A \Rightarrow B$ and $\overline{B} \Rightarrow \overline{A}$ (Law of contraposition), constructing of deductive reasoning.

Tool use: PowerPoint, Smart Board, regulatory cards, pen and paper, chalk and blackboard.

Assignment 5: Evaluate

Learning objective: Assess how students understand the explored phenomenon.

Student activity: Solving similar tasks, testing, reflection.

Tool use: PowerPoint, Smart Board, regulatory cards, Padlet interactive board, Kahoot, pen and paper, chalk and blackboard.

Suggestions for use

The unit can be changed by choosing another prompt linked to scheme of learning (for example, from algebra).

It is possible to offer the following additional ways and methods of stimulation and motivation interest in student learning:

- use of the surprise effect (choose a prompt that you think will intrigue the class);
- heuristic conversation;
- creation of a situation for free choice of the study task by students, etc.

C. Worksheet and files

A PLATINUM unit contains
- *supplementary files*
  Regulatory cards (www.inquirymaths.org)