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Case problem: "To define the limit of sequence"
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Course	Mathematical Analysis	**Activity**
Year of study	First	
Topic	The concept of limit of sequence	
Purpose of the case	Formation of concepts using the technology IBL	
Case description	*Engagement* The examples of familiar (from previous 7 - 10 lessons) numerical series are given, where only 3 are convergent (i.e, $x_n = \frac{1}{n}$, $y_n = \frac{(-1)^n}{n}$, $z_n = \frac{n}{n+1}$). It is announced, that these 3 sequences are characterized in Mathematics as convergent to 0, 0 i 1, accordingly. Others, on the contrary, are not convergent, and it is said they are divergent. It is proposed to students, based on observations, to observe the characteristic properties of the convergent sequence and to give its definition.	*Teacher:* - provides conditions for observation (possibility to use computers with access to the Internet); *Students:* - understand the essence of the task and the need to gain new knowledge
	Exploring Using dynamic models, observing the behavior of members of convergent and divergent sequences with an increase of their serial number, the answers to important questions are coming: - how do the members of convergent sequences behave if their serial numbers increase? (approaching to a certain point of a numerical line); What about divergent ones? (no tendency to approach to specific point; there are either no such points at all, or there is more than one, or if there is one such point at which the members of the sequence are "gathered", then the other "group" (also infinite), on the contrary , "flee" from it); - how close do the members of the convergent sequence approach to that particular point? (they are thickening near that point, that is, they fit any way close) In this way, the students will get the formed concept of the limit of sequence on the intuitive (sensory) level and in the geometric representation.	*Teacher:* - leads in the right direction of observation, motivating and directing students to pay attention to those features that are characteristic for the convergent sequence; - helps, opposes (if necessary brings counterexamples); - at this stage needs to make students be able to precisely characterize the convergent sequence and explain why all of given earlier sequences, except for the 3 specified were not convergent "in their own words", without strict mathematical formulas, but using geometric interpretations *Students:* - construct, for example, in GeoGebra, and examine and analyze dynamic models of convergent and divergent sequences; - find differences in these

		two groups of sequences
	Explanation Accurate mathematical interpretation of descriptive explanations: "members of the sequence are approaching to the point any way close," "it is impossible that infinite number of members of sequence are far from that point," etc.	*Teacher:* - by guiding questions helps students to make this "translation"; *Students:* - actively working on forming the ability to adequately use mathematical language and symbols, precisely use appropriate terms.
	Elaboration A. Geometric definition of the limit of sequence. B. Definition of sequence in terms of mathematical analysis. C. Work on a conscious, complete and profound understanding of the notions; remembering the definitions. D. Record the definitions by means of mathematical symbols.	*Teacher:* - helps (if necessary) formulate definitions, encouraging them to do so on their own; - asks students a question: what will happen when in the definition of the limit of sequence, instead of "for any ε> 0" to say "exists ε> 0?", seeks to get a deep understanding of the notions of this concept; encourages students to ask similar questions and answer them. *Students*: - under the direction of the teacher actively work on improving the structure of their newly formed knowledge
Expected result	- formation of investigation skills; - development of logical thinking; - development of the ability to observe and allocate on its basis the essential features of the phenomenon under study; - development of communicative qualities; - development of mathematical language; - gaining by the students, as a result of investigation activities, new (to them) knowledge.	*Evaluation* A. Is the problem completely resolved? B. Do we have a clear idea of the convergent (divergent) sequence? C. Can we formulate the definition of the limit of sequence? Can we write it down with the help of mathematical symbols?
Number of hours	1 academic hour	
Using digital technologies	Computers, Internet access, smart board	
Other equipment	Chalk, board	