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Department of Computer Science and Mathematics

Case study (research, for the formation of new knowledge)
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<tr><td>Course</td><td>Mathematical Analysis</td><td rowspan="4">Activity</td></tr>
<tr><td>Year of study</td><td>First</td></tr>
<tr><td>Topic</td><td>Absolute and conditional convergence of a numerical series</td></tr>
<tr><td>Purpose of the case</td><td>Organization of studying using the technology IBL</td></tr>
<tr><td rowspan="3">Case description</td><td>Preliminary base of knowledge and skills:
- notion of a numerical series, its convergence / divergence;
- properties of converging a series, in particular, the necessary condition of convergence;
- Cauchy criterion of convergence;
- signs of convergence of a positive series;
- Leibniz criterion of convergence of an alternating series;
- ability to investigate the convergence of a series and, in case of convergence, to find the sum using the definition;
- understanding of what is the sufficient condition (sign); necessary condition;
- understanding of cases in which using the necessary condition of the convergence of a series can be effective and ability to apply it on practice;
- formed skills of investigating the convergence of positive and alternating series using sufficient condition.</td><td>Teacher: (15 min.):
Actualization of prior knowledge and skills in the form of frontal conversation with maximum involvement of all students; in addition to theoretical questions, exercises for oral investigation of convergence of a series or finding out what criteria should be used for the investigation of convergence of a series are proposed.
Students:
- active participation in training</td></tr>
<tr><td>Engagement
The problem is proposed: to investigate the convergence of not positive series (contains both positive and negative members).</td><td>Teacher: creates conditions in which a student must recognize (see, feel) the need for new knowledge
Students: Understand that they have not enough knowledge to solve the problem</td></tr>
<tr><td>Exploring and Explanation

Research, analysis, reasoning, search of a way of solving a problem; hypothesis formulation.

Approximate step by step reasoning
A. The series is not positive, and we have (at this point) in the tool arsenal - signs of convergence of positive rows.</td><td>Teacher:
- plays the role of facilitator in discussing the problem and ways of its solving: helps, directs, discuses, opposes (if necessary, gives counterexamples, asking questions that push the student to the correct resolution)</td></tr>
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	B. Would it be possible to use these signs to investigate not positive series? (Key problem). The idea is to turn to finite sums. If we have the sum of a **finite number** of items, then it is obvious that when we change the sign "+" to "-" for part of the items, the sum will decrease. Common sense suggests that the same will be in the series: if a positive series is convergent, then the replacement of a part of its members by opposite members can only reduce the sum of a series, that is, the series will remain convergent. This suggests an affirmative answer to the question. C. The hypothesis is formulated: "If a series, formed from the modules of the members of this series, is convergent, then this series is convergent". D. Search for the idea of proof the hypothesis	*Students:* - analyze, think, discuss, discute, ask questions that help to advance in the research, express ideas and oppose; - formulate the hypothesis - as a result of the discussion come to the method of proof – check the Cauchy criterion of the convergence of a series
	Elaboration A. Proof of the formulated hypothesis using the Cauchy criterion. B. Formulation of the theorem. C. Determining whether a proven sign is necessary. D. Definition of an absolutely (conditionally) convergent series. E. Formulation and recording on the board of the topic of the lesson. F. Solving problems on investigation of the convergence of not positive series	*Teacher:* - writes (while conducting a conversation with the students, taking into account their comments) the proof of the theorem on the board; - serves as a facilitator when discussing a new problem: what can one say about this series if the series made of modules of its members is divergent? - formulates the definition of absolute and conditionally convergent series; - offers training exercises; *Students:* - provide "help" to the teacher in the process of proof; - write in the notebook (together with the teacher on the board) the formulation and proof of the theorem about the absolute convergence of the series; - on the basis of the discussion come the assumption that the proved sign is not necessary. Provide an appropriate example (independently or with the help of a teacher).
	Exploring (Continuation)	

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Derivative problem («case in case»)
It is proposed to continue the investigation of the solved problem.
Teacher:
"We have already established that from the convergence of the series
$$\sum_{n=1}^{\infty}|a_n| \qquad (2)$$
follows the convergence of series
$$\sum_{n=1}^{\infty}a_n\,. \qquad (1)$$
If the series (2) is divergent, then the series (1) can be both convergent or divergent. We have also investigated a large number of series (both collectively and each his or her "own" series) on absolute convergence, using the proved theorem. I propose to analyze the obtained results using the scheme:
<table>
<tr><th>Convergence of series (2)</th><th>Sign, used to investigate the convergence of series (2)</th><th>Convergence of series (1)</th></tr>
<tr><td>+</td><td></td><td>+</td></tr>
<tr><td>–</td><td></td><td>+</td></tr>
<tr><td>–</td><td></td><td>–</td></tr>
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Based on the analysis, give a hypothetical answer to the question: "In which case it can be stated that the divergence of the series (2) follows from the divergence of the series (1)?" Check (prove or disprove) the hypothesis. If the hypothesis is confirmed, then what practical value it might have? "
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Teacher:
- proposes continuation of the investigation on the basis of the experiment;
- encourages the expression of the hypothesis;
- during the discussion, performs the role of senior colleague, assistant
Students:
- collectively fill in the table proposed by the teacher, analyze the received results, express hypotheses, discuss, oppose;
- form a "coherent" hypothesis (If the divergence of the series (2) was established on the basis of d'Alembert or Cauchy criteria, then it can be argued that the series (1) is divergent);
- propose its proof
- find out the practical significance of the knowledge gained; give the relevant examples.
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<td>Expected result</td>
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- formation of investigation skills;
- development of logical thinking;
- development of communicative qualities;
- as a result of investigating activities obtaining by the students new (to them) knowledge
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Evaluation
A. Are the problems encountered completely resolved?
B. What was learned?
C. How can this be applied (what new problems can be solved now)?
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<td>Number of hours</td>
<td>2 academic hours</td>
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<td>Using digital technologies</td>
<td>Smart board at the stage of actualization knowledge and skills</td>
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<td>Other equipment</td>
<td>Chalk, board</td>
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